

Contents

Short Guide to the Book	9
What is This Book About?	9
Where to See the Art Objects Discussed in the Book?	9
Audience	9
Navigation	10
Acknowledgments	11
Chapter 1. Three Letters M	13
1.1 M is for Manhattan	13
1.1.1 In the heart of the country	13
1.1.2 How Manhattan was founded	14
1.1.3 The long road to a college town	14
1.2 M is for Museum	16
History of the Marianna Kistler Beach Museum of Art by Kathrine Schlageck	17
1.3 M is for Mathematics	18
1.4 How It Worked for Us: Math and Art Workshops at the Beach Museum of Art	21
Chapter 2. Regular Polyhedra	25
2.1 About the Artwork [<i>Kansas Meatball</i>]	25
2.2 Polyhedra	26
2.3 About the Artist [Alan Shields]	27
2.4 Other Examples in Art	28
2.4.1 Polyhedra	28
2.4.2 Geodesic domes	29
2.5 Why Are There only Five Platonic Solids?	30
2.6 Euler Characteristic. Magic of the Number 2	33
2.7 Dual Polyhedra	34

2.8	Math Problems	34
2.9	Art Projects	35
2.9.1	Paper plate icosahedron	36
2.9.2	Toothpick icosahedron	39
2.10	Answers	39
Chapter 3. Combinatorics and Probability		41
3.1	About the Artwork [<i>21 with Cube</i>]	41
3.2	What is a Die?	42
3.3	Probability Theory. It All Started with a Game	42
3.4	About the Artist [Jesus Manuel Montes]	44
3.5	Card Games and Games of Chance in Art	45
3.6	Randomness as Art Technique	46
3.7	Math Problems	47
3.8	Probability Experiments in Class	51
3.9	Art Projects	53
3.9.1	Decorated die	53
3.9.2	Random Person	54
3.10	Answers	57
Chapter 4. Symmetry		61
4.1	What is Symmetry?	61
4.2	Frieze Patterns	64
4.3	Symmetries of Frieze Patterns	69
4.4	Seven Types of Frieze Patterns	70
4.5	About the Artists	71
4.5.1	Jack Chevalier	71
4.5.2	Pueblo communities	72
4.6	Math Problems	74
4.7	Art Projects	74
4.7.1	Paper People	74
4.7.2	Decorate a vase	74
4.8	Answers	74
Chapter 5. Circles		75
5.1	Clay Painting with Circles	75
5.2	About the Artist [Orval F. Hempler]	76
5.3	Other Examples in Art	77
5.4	Wonderful Circles around Us	77

5.5	Circles and Right Angles	79
5.6	More Problems about Circles	83
5.7	Art Projects	84
5.8	Answers	85
Chapter 6. Fibonacci Sequence		89
6.1	About the Artwork [<i>Pyrrho Techniko</i>]	89
6.2	Pattern of the Table	90
6.3	Fibonacci Numbers	91
6.4	Golden Ratio as a Limit of Fibonacci Sequence	92
6.5	About the Artist [John L. Vogt]	92
6.6	Some Words of Caution from M. Gardner, G. Markowsky, C. Falbo, U. Eco, et al.	94
6.7	Phyllotaxis and Examples of the Golden Ratio and Fibonacci Numbers in Modern Art	97
6.7.1	Pinecones	98
6.7.2	Modulor	98
6.7.3	Other examples	100
6.8	Geometric Construction of Golden Ratio	101
6.9	Problems on Properties of Fibonacci Numbers	102
6.10	Playing with Sequences	104
6.11	Answers	105
Chapter 7. Geometrical Optics		109
7.1	About the Artwork [<i>Konza Prism</i>]	110
7.2	Travel of Light	110
7.3	Artist's Choices	112
7.4	About the Artist [Roger Lane Routson]	113
7.5	Other Examples in Art	113
7.6	Snell's Law	113
7.7	The Reflection Principle and Billiards	117
7.8	Refraction Experiment	120
7.9	Art Project [Rainbow Paper Collage]	120
7.10	Answers	122
Chapter 8. Linear Perspective		127
8.1	Space Presentation	127
8.2	Main Features of Linear Perspective	128
8.3	Picture Plane and Ground Plane	129

8.4	Horizon Line	130
8.5	Vanishing Points	132
8.6	The Number of Vanishing Points	136
8.6.1	No vanishing points: front view	136
8.6.2	One-point perspective	137
8.6.3	Two-point perspective	137
8.6.4	Multi-point perspective	138
8.7	Do Artists Rigorously Follow Mathematical Rules of Linear Perspective?	139
8.7.1	<i>Anderson Hall</i>	140
8.7.2	<i>Spring and Mercer</i>	140
8.8	Why Linear Perspective Rules Are not Always Followed by Artists	141
8.9	About the Artists	143
8.9.1	Charles Merrick Capps	143
8.9.2	Stevan Dohanos	144
8.9.3	Norma Bassett Hall	144
8.9.4	George M. Kren	144
8.9.5	Roy Langford	145
8.9.6	Carol Pylant	145
8.9.7	Larry W. Schwarm	147
8.9.8	Shirley Smith	147
8.10	Problems on Linear Perspective	148
8.11	Answers	152
Chapter 9. Star Polygons		155
9.1	About the Artwork [Lead Crystal Vase]	155
9.2	Definition of Star Polygons	155
9.3	Star Polygons with Different Step Lengths	157
9.4	The Greatest Common Divisors	157
9.5	About the Artists [Rückl Crystal]	158
9.6	Art Projects	159
9.6.1	Decorated plate	159
9.6.2	Vase drawing	159
9.6.3	String star polygon	160
9.7	Proof of Proposition 9.1	162
9.8	Answers	164

Chapter 10. Topology **165**

10.1 About the Artwork [*Klein Bottle*] 165

10.2 Nonorientable Surface 165

 Klein bottle = $2 \times$ Möbius band..... 168

10.3 About the Artist [Mitsugi Ohno] 168

10.4 Other Examples in Art 169

10.5 Topological Projects 170

 10.5.1 Let us cut. 170

 10.5.2 Klein bottles and tori from pipe cleaners 170

10.6 Topology 172

10.7 Problems 173

10.8 Tracing Designs 174

10.9 Knots 176

10.10 Answers 177

References **179**

Artwork Index..... **185**

Image Index **189**

Name Index..... **195**

Subject Index..... **197**