SHORT GUIDE TO THE BOOK

What is This Book About?

This book is devoted to timeless bonds between art and mathematics, which are illustrated by artworks from the collection of the Marianna Kistler Beach Museum of Art. The museum is located on the campus of Kansas State University, in the small and lovely town of Manhattan, Kansas. Each chapter of the book starts with a study of an art object from the collection. The emphasis is placed on a pattern structure, symmetry, some mathematically inspired content, or a mathematically motivated technical solution in the production of the object. The discussion contains facts, proofs, problems, solutions, as well as math-related educational art projects.

Where to See the Art Objects Discussed in the Book?

The reader should not be concerned if he or she never had a chance to visit Manhattan, Kansas: all artworks mentioned in this book from the Marianna Kistler Beach Museum of Art are displayed online in the digital collection of the museum:

www.beach.k-state.edu/explore/collection/

Moreover, masterpieces discussed in the book are very far from being the only examples of art carrying mathematical features. It is quite possible that the reader's local art museum holds similar examples in its collection; for example, linear perspective and symmetry patterns can be seen in almost any art museum. And moreover, for the reader's convenience, some renown examples *from the collections of the most prominent museums of the world* are listed in almost every chapter. Their reproductions are widely distributed in printed catalogues and online.

Audience

The book is addressed to readership in a broad spectrum of ages and professional interests. We hope that a person interested in relations between art and mathematics will find something curious, new and challenging in the book. Discussions are enhanced with numerous entertaining illustrations. Not all, but certainly many parts of the book can be comprehended by young readers on their own.

One of our goals was to make the book useful to readers who may want to use it to teach other people about art and mathematics. Thus, the specific format of the book was dictated by possible needs of these particular readers: parents, art teachers, math teachers, math circle instructors ... Moreover, materials presented here originate from real math and art workshops. They took place in the framework of a math enrichment program at Kansas State University (for more details see Section 1.4).

Navigation

Each chapter can be read independently. Most chapters are structured as follows.

- An art object from the Marianna Kistler Beach Museum of Art with a connection to a particular topic in mathematics is introduced.

- A short discussion of the properties of the selected art object is followed by a review of related topics in mathematics.

- Creative projects and problems of different levels of difficulty complement the review. Answers and solutions are provided at the end of the chapter.

- Whenever it is possible, we share information about the artist that created the selected artwork.

- Most chapters contain lists of additional examples of artworks related to the mathematical topic of the chapter. These famous artworks belong to collections of world renown museums. Their reproductions can be found online and in art catalogues.

Problems in the book vary in their levels of difficulty, and for the reader's convenience we mark the levels with symbols:

 labels easy short questions,

 is for intermediate level problems, and

 marks problems of the most challenging level in this book.

Note that many easy short questions can be used as well as a warm-up at lessons for advanced students.

Acknowledgments

This project was accomplished with assistance of many professionals.

Of course, this book would not be possible without valuable contributions from the staff of the Marianna Kistler Beach Museum of Art.

I would like to thank the Registrar and Collections Manager Sarah Price, the Assistant Registrar Theresa Marie Ketterer, the Curator Elizabeth Seaton.

My special thanks are due to the Director Linda Duke for the indispensable support during the preparation of the manuscript and the Senior Education Specialist Kathrine Schlageck for many years of fruitful collaboration on math and art workshops for our math circle program.

I would like to thank the copyright holders of the images used in the book for their kind permission of reproduction.

I would like to express my gratitude for professional consultations of Dr. Lado Samushia (Department of Physics at Kansas State University), the Curator of Archives and Librarian Linda Glasgow (Riley County Historical Society and Museum Research Library and Archives), Dr. Nadia Sidorova (Department of Mathematics at University College London).

Valuable comments by my husband Dr. Ilia Zharkov (Department of Mathematics at Kansas State University) and my son Styopa Zharkov brought a fresh angle on some mathematical interpretations discussed in the book.

While working on the biographies of the artists, I spent many hours absorbed in the treasures of the University Archives and Manuscripts in the Department of Special Collections of Kansas State University. I would like to recognize the highly professional and friendly help of the staff of the Archives.

I am very grateful to my esteemed colleague, the determined editor Professor Robert Burckel for the time and effort he spent on improving the manuscript.

The whole project originates in the sessions of Math Circle Seminar at Kansas State University. Many years of shining curiosity and outstanding thirst for knowledge of the participants inspire me and my colleagues to teach mathematics to younger generations. I would like to thank all the students of Math Circle Seminar and their parents for the enjoyable long-lasting experience of communicating math and its applications.

This book is my second collaborative project with my mother, Tamara Rozhkovskaya. The first book, entitled *Math Circles for Elementary School Students* [1] turned out to be a very successful title, very well accepted by both Russian- and English-speaking readers.

As with the first book, it is not possible to express the full extent of my gratitude to my mother for her dedication, encouragement, time and efforts that she put in this project. Through many hours of polishing and perfecting, she brought the project from the state of a draft to a real book. I am very grateful to her for this magic. And another magic happened here: we live very far from each other in opposite parts of the planet, but this collaboration miraculously shortened the distances that separate us. Just for this wonderful reason it would be worth writing a book like this.

Natasha Rozhkovskaya
Manhattan KS, USA
May 8, 2016

[1] Natasha Rozhkovskaya, *Math Circles for Elementary School Students* [in Russian], Tamara Rozhkovskaya Publisher, Novosibirsk (2011); English transl.: MSRI & Am. Math. Soc., Providence, RI (2014).

Chapter 2

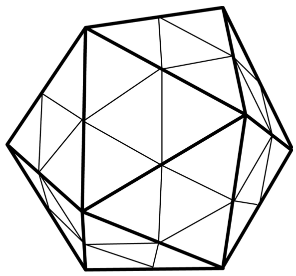
REGULAR POLYHEDRA

FIGURE 2.1. *Kansas Meatball* by Alan Shields. KSU, Beach Museum of Art.

2.1 About the Artwork [*Kansas Meatball*]

Look at the pair of huge nested geometrical structures floating over the heads of museum visitors. The acclaimed artist Alan Shields gave the playful name *Kansas Meatball* to his creation. Shields' good sense of humor is traced in many titles of his works. He always considered the evocative titles to be an important part of his art [**10**, p. 80].

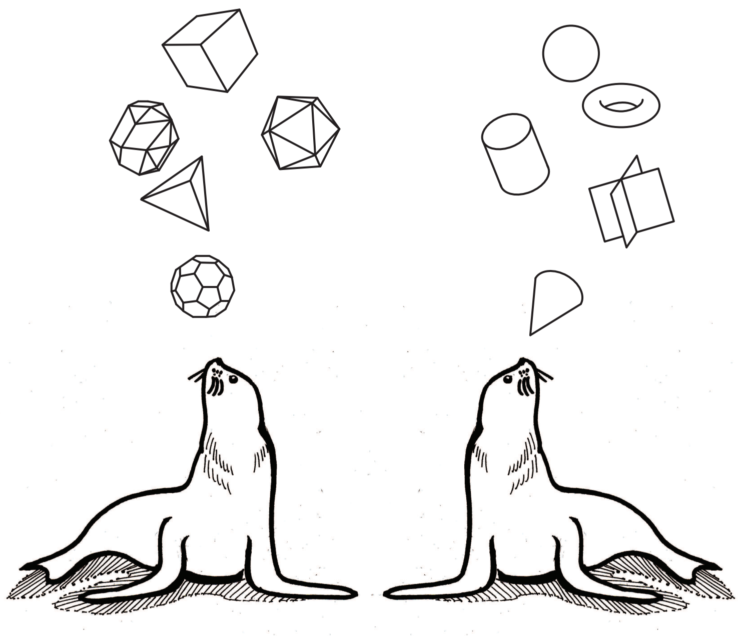
Meanwhile, the symmetries of the structure make an open invitation to study *Kansas Meatball* from the point of view of geometry. Let us accept the invitation and investigate the shape of the outer structure of *Kansas Meatball.* It is made of regular triangles. They can be arranged in groups of four triangles that lie in the same plane. The four triangles of one flat group make a bigger regular triangle, called a *face*. The outer shell of the sculpture consists of equal faces that are regular (flat) triangles. The geometric shape they create is loved by mathematicians since ancient times. It is called an *icosahedron* and is an example of a regular polyhedron.



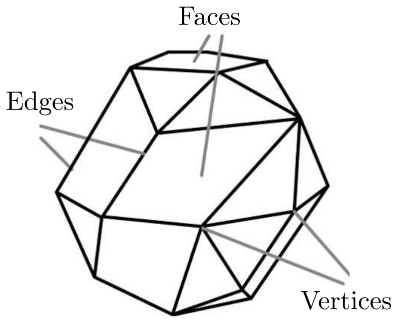
The outer shell of *Kansas Meatball* is an icosahedron.

2.2 Polyhedra

Let us discuss in more detail the above-mentioned mathematical notions. A *polyhedron* is a closed three-dimensional solid bounded by polygons.



For example, the solids on the left are polyhedra and the solids on the right are not polyhedra.

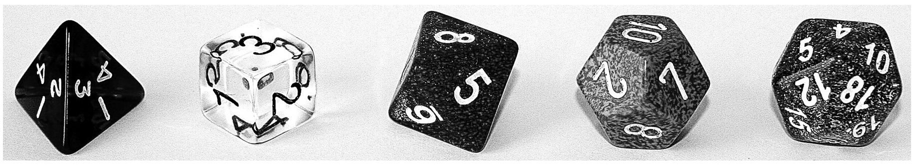


The polygons that create the boundary of a polyhedron are called *faces*. Two neighboring faces of a polyhedron are joined along a segment of a line, called an *edge*. If two or more edges have a common point, that point is called a *vertex*. Roughly speaking, vertices are "corners" of a polyhedron.

Some polyhedra (for example, a cube and an icosahedron) are symmetric. Actually, there is a family of polyhedra, called *regular polyhedra* or *Platonic solids*, that are "perfect" from the point of view of symmetry. *Platonic solids* are convex polyhedra satisfying the following two defining conditions.

- All the faces of a Platonic solid are equal *regular polygons*, all of the same shape and size.
- The number of faces meeting at any vertex is the same for all the vertices.

It turns out that these two conditions imply that *there exist only five Platonic solids: A tetrahedron, a cube, an octahedron, a dodecahedron, and an icosahedron* (see the proof of this beautiful fact in Section 2.5).



2.3 About the Artist [Alan Shields]

Alan Shields (1944–2005) was born in Herington, Kansas. He spent his childhood on his family's farm and later attended school in Salina. In 1963–1966, Shields was enrolled in the civil engineering program at Kansas State University, where at some point he made a decision to change his life and to pursue a career in art. In 1968, Shields left the university without graduating, and headed to New York City. In 1972, the artist moved to a house on Shelter Island situated in the bay at the eastern end of Long Island, where he resided and worked until his death.

Shields worked with a very wide range of materials and media. One of Shields' signature design elements is application of sewing machine stitches. He learned to sew as a child. In one of the interviews **[11]**, the

Chapter 6

FIBONACCI SEQUENCE

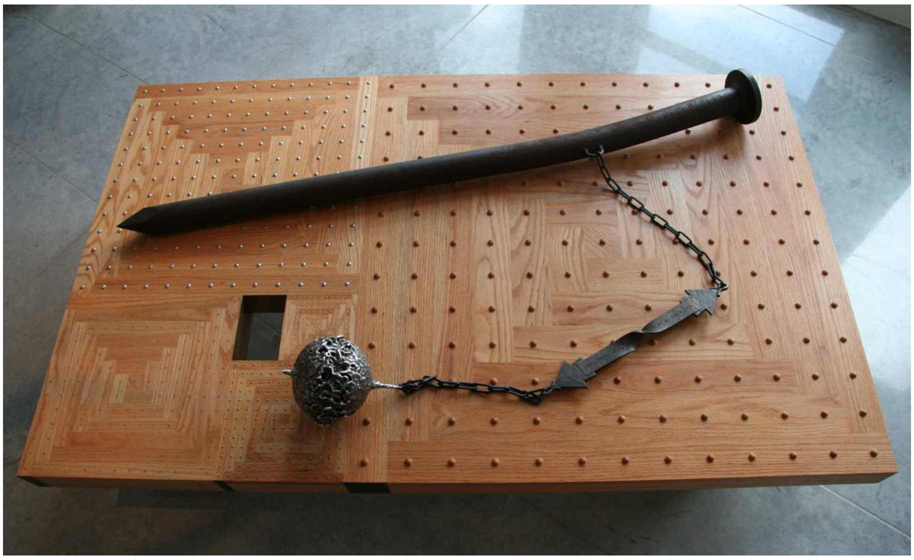


FIGURE 6.1. *Pyrrho Techniko* by John L. Vogt.
KSU, Beach Museum of Art.

6.1 About the Artwork [*Pyrrho Techniko*]

A huge nail, chained with a ball, is placed on the decorated top of a wooden table. One can easily visualize that the objects were dropped by some absent-minded giants who may soon come back for their belongings. The words "Nothing in itself is more this than that" on the arrow-shaped plate attached to the chain add to the mystery of the artwork by John L. Vogt. We quote the words of his former colleague Charles Stroh [**46**]:[14]

> "... He made objects in steel and stainless steel that he combined with a variety of materials resulting in humorous or playful juxtapositions. He would work hours and hours creating something that would appear to possibly have a function or maybe had a function, but what that function would be was unclear. He played with ambiguity both in his use of materials an in the objects themselves..."

[14] Charles Stroh was a professor (1980–1997) and the Head of the Department of Art (1980–1989) at Kansas State University.

Among all the mysteries of this beautiful sculpture, there is one that can be a perfect topic for mathematical discussion: the pattern of the decorated tabletop. It is clear that there is a rule of placing squares in the pattern, and the rule is just astonishing.

6.2 Pattern of the Table

To understand the rule, let us reproduce the diagram of the tabletop pattern. In Figure 6.2, the dimensions of the rectangle are proportional to the dimensions of the tabletop and the black rectangle corresponds to the hole.

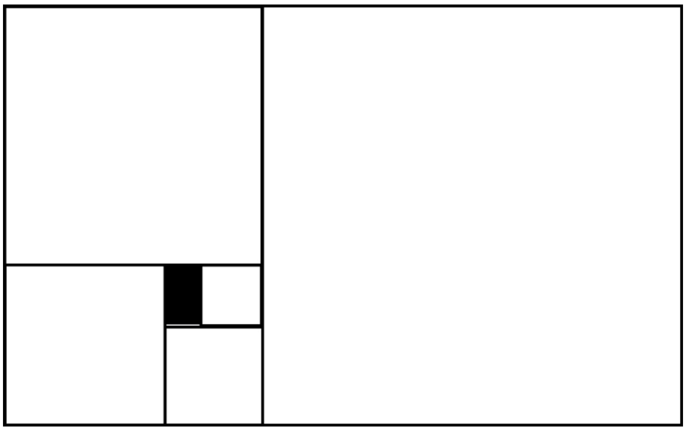


FIGURE 6.2.

The size of the table is $160 \times 99\ \mathrm{cm}^2$. The sizes of the sides of the squares of the pattern form the sequence

$$99 \text{ cm}, \quad 61.1 \text{ cm}, \quad 37.8 \text{ cm}, \quad 23.3 \text{ cm}, \quad 14.3 \text{ cm}, \quad 9 \text{ cm}. \tag{6.1}$$

Consider the golden ratio $\varphi = (1+\sqrt{5})/2$ which is an irrational number with the approximate decimal value

$$\frac{1+\sqrt{5}}{2} \simeq 1.6180339887+. \tag{6.2}$$

Then the sequence (6.1) can be compared with the sequence

$$\frac{160}{\varphi} \text{ cm} \simeq 98.885 \text{ cm}, \quad \frac{160}{\varphi^2} \text{ cm} \simeq 61.115\text{cm} \quad \frac{160}{\varphi^3} \text{ cm} \simeq 37.771 \text{ cm},$$
$$\frac{160}{\varphi^4} \text{ cm} \simeq 23.344 \text{ cm}, \quad \frac{160}{\varphi^5} \text{ cm} \simeq 14.427 \text{ cm}, \quad \frac{160}{\varphi^6} \text{ cm} \simeq 8.916 \text{ cm}.$$

The placement and the relative sizes of the squares suggest that this artwork intentionally refers to the golden ratio. Thus, the artwork *Pyrrho Techniko* invites us to review properties of this famous number and the related legendary sequence of Fibonacci numbers.

6.3 Fibonacci Numbers

We list the first few elements of the sequence of Fibonacci numbers:

$$1,\quad 1,\quad 2,\quad 3,\quad 5,\quad 8,\quad 13,\quad 21,\quad 34,\quad \ldots$$

The defining rule is the following: *Any number of the sequence is the sum of the preceding two numbers* or, equivalently,

$$F(n) = F(n-1) + F(n-2), \quad n = 3, 4, 5, \ldots \tag{6.3}$$

with the initial condition $F(1) = F(2) = 1$. Formula (6.3) is a *recurrence relation.* It allows us to find the next term provided that we know the preceding two.

Problem 6.1. We know that $F(19) = 4181$ and $F(20) = 6765$. Find $F(21)$.

If we do not know the preceding values, we still can find the value of $F(n)$ with the help of the *closed formula.* This formula depends only on n and has the form

$$F(n) = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right). \tag{6.4}$$

The proof of this formula can be found in many standard textbooks on combinatorics (see, for example, [**47**] or [**48**]).

Note the amazing property of formula (6.4). This formula involves complicated expressions with square roots of 5, but the answer is always a positive integer number, without any roots or fractions. For example, check that

$$\frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^2 - \left(\frac{1-\sqrt{5}}{2}\right)^2\right) = 1,$$

$$\frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{3}\right)^3 - \left(\frac{1-\sqrt{5}}{2}\right)^3\right) = 3,$$

$$\frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^6 - \left(\frac{1-\sqrt{5}}{2}\right)^6\right) = 8.$$

6.4 Golden Ratio as a Limit of Fibonacci Sequence

The number $\varphi = (1+\sqrt{5})/2$ in formula (6.4) is the famous *golden ratio*. It is often introduced as the ratio of lengths of sides of a golden rectangle.

Definition 6.1. A *golden rectangle* is a rectangle whose length a and width b satisfy the relation

$$\frac{a+b}{a} = \frac{a}{b}. \tag{6.5}$$

Problem 6.2. Prove that if a and b satisfy the relation (6.5), then

$$\frac{a}{b} = \frac{1+\sqrt{5}}{2}.$$

Fibonacci numbers are closely related to the golden ratio. Formula (6.4) not only contains this famous number, but also allows one to prove the following statement [**48**]: *When n grows to infinity, the ratios of two consecutive Fibonacci numbers tend to the golden ratio*

$$\lim_{n\to\infty} \frac{F(n)}{F(n-1)} = \frac{(1+\sqrt{5})}{2}, \tag{6.6}$$

which means that for large enough n the values of the ratio $\dfrac{F(n)}{F(n-1)}$ are very close to the golden ratio value. For example, look at the decimal values of the first few ratios of Fibonacci numbers and compare them with the approximate decimal value of the golden ratio (6.2):

$$\frac{F(2)}{F(1)} = \frac{1}{1} = 1, \qquad \frac{F(3)}{F(2)} = \frac{2}{1} = 2, \qquad \frac{F(4)}{F(3)} = \frac{3}{2} = 1.5,$$

$$\frac{F(5)}{F(4)} = \frac{5}{3} \simeq 1.667, \qquad \frac{F(6)}{F(5)} = \frac{8}{5} = 1.6, \qquad \frac{F(7)}{F(6)} = \frac{13}{8} = 1.625,$$

$$\frac{F(8)}{F(7)} = \frac{21}{13} \simeq 1.6154, \qquad \frac{F(9)}{F(8)} = \frac{34}{21} \simeq 1.6190,$$

$$\frac{F(10)}{F(9)} = \frac{55}{34} \simeq 1.6176 \quad \ldots$$

6.5 About the Artist [John L. Vogt]

John L. Vogt (1930–2009) was born in Norbon, Missouri. He grew up in the Kansas City area with two brothers and a sister. In 1950–1953, Vogt served in the US Army Infantry in Okinawa (Japan). A few years later after honorable discharge from military service, Vogt took his path into an art career. In 1961, he received a Bachelor degree of Fine Arts from the Kansas City Art Institute, where he was recognized as the highest ranking senior. Two years later, Vogt completed the Master of Fine Arts degree at the University of Illinois. The same year, he became a faculty member of the Department of Art at Kansas State University, where he taught sculpture until his retirement in 1991 [**49**].

Besides dry official information reproduced above, one can find in the university archives some memories about the artist with a human touch that give a clear hint of his generous personality. It seems that, during the first years at Kansas State University, good and bad fortune followed the sculptor. In the first year of employment, Vogt agreed to exhibit his work in the Union Art Lounge at Kansas State University. This was a last-minute substitution made by the organizers of the exhibition: another artist, scheduled to exhibit his work, declined the invitation since the previous year someone smeared wet paint on his paintings and completely damaged the artwork. And bad luck followed this series of exhibitions again: less than 24 hours after the exhibit opened in the Union, one of Vogt's collages was stolen [**50**]. Other works of Vogt, remaining at that unlucky exhibition, were two bronze sculptures created by the lost-wax casting (*cire perdue*) process. This is a method of casting bronze, in which a mold is formed around a wax model. The wax model is subsequently melted and drained away. And the fate of Vogt's bronze sculptures was far more fortunate: only a year after that unhappy incident at the Union, Vogt's bronze sculpture *A Figuration from Wax* was chosen for the XIVth Annual Mid-America Art Exhibition at the Nelson-Atkins Museum of Art in Kansas City. The competition was very selective: the jury of that event reviewed 1,160 works and only 80 from those were chosen. The following modest comment made by the artist on that occasion reveals his integrity and impartiality as a judge of art:

> "Many of the submitted works were of the exhibition quality and possibly with another set of judges a different 80 would have been chosen. The criteria for judging art are not clear although there are well known standards which judges follow." [**51**]

During subsequent years, Vogt participated in numerous exhibitions nationwide. For many years he was also actively involved in the beautifying of the Kansas State University campus. Vogt supervised three summer workshops at the end of the sixties that gathered students in art, architecture, and landscape architecture:

> "The students made proposals for several areas of the campus which were, at the time, in pretty poor condition. The better ones and those that were financially feasible were selected and constructed, with the students doing most of the work. Some were sculptures, others involved architecture. The idea was to integrate the three different disciplines of art, architecture and landscape architecture. There were no titles given to any of the works and Vogt said he could not name the emanation of any of the projects." **[52]**

Even though the art objects were intended to be placed temporarily, the traces of the project enhanced some campus areas for many years [**53**].

6.6 Some Words of Caution from M. Gardner, G. Markowsky, C. Falbo, U. Eco, et al.

The pattern of the table indicates that Vogt intentionally referred to the golden ratio in his artwork. The reader may expect that now we will list standard examples: ancient Greek architecture, works of Leonardo Da Vinci, shells, petals of flowers, etc. But no, we will not. And the reason is very simple: in spite of widely repeated citations of these "examples," most of them are very inadequately justified.

It is in human's nature to seek for harmony in the universe. We admire beautifully stated laws of physics, magnificent formulas in mathematics, and balanced relations of science. The golden ratio is often cited as an example of a harmonious connection between math, art, and nature. So much is written about the appearance of the golden ratio in art and nature, that it comes as a surprise that from the point of view of rigorous science, unfortunately, significantly many of these "standard examples" are questionable.

George Markowsky describes the general situation as follows.

G. Markowsky: "Generally, its mathematical properties are correctly stated, but much of what is presented about it in art, architecture, literature, and esthetics is false or seriously misleading. Unfortunately, these statements about the golden ratio have achieved the status of common knowledge and are widely repeated." [**54**, p. 2]

We invite the reader to develop a critical approach to the subject of appearance of the golden ratio in art and nature and to build his or her own opinion about the true extent of its role. There exist a number of publications, where the authors separate valid statements from general misconceptions. But it is hard to hear those individual skeptical voices in the chorus of "well known facts" that are repeated in the literature without really looking into these "facts." We would like to refer the reader to the following papers:

Martin Gardner, "The cult of the golden ratio" [**55**].

George Markowsky, "Misconceptions about the golden ratio" [**54**].

Clement Falbo, "The golden ratio – a contrary viewpoint" [**56**].

Roger Herz-Fischer, "The home of golden numberism" [**57**].

Let us review some of the questionable statements and outline the reasons for skepticism. For more details we refer to [**55**]–[**57**].

- **Fibonacci numbers appear in nature as numbers of petals.**

Flowers with three or five petals are common. But flowers with four and six petals are frequently found too, and these are not Fibonacci numbers. Therefore, it is not clear, why Fibonacci numbers should be recalled in this case – they are no more relevant to the number of petals than any other numbers.

- **Sea shells grow according to the so-called golden spiral, which can be inscribed into a sequence of golden rectangles.**

We refer to [**56**, p. 127] (below, φ denotes the golden ratio):

C. Falbo: " In 1999, I measured shells of Nautilus pompilius, the chambered nautilus, in the collection at the California Academy of Sciences in San Francisco. The measurements were taken to the nearest millimeter, which gives them error bars of ± 1 mm. The ratios ranged from 1.24 to 1.43, and the average was 1.33, not φ (which is approximately 1.618). Using Markowsky's $\pm 2\%$ allowance for φ to be as small as 1.59, we see that 1.33 is quite far from this expanded value of φ. It seems highly unlikely that there exists any nautilus shell that is within 2% of the golden ratio, and even if one were to be found, I think it would be rare rather than typical."

Probably, this general misconception originates in confusion between a general logarithmic spiral and a particular case of a logarithmic spiral, named a *golden spiral*. Indeed, there are many examples of spirals in nature. The growth of some of them is well modeled by mathematical functions such as logarithmic spirals. Logarithmic spirals are curves of the form $r(t) = r_0 a^t$ in the polar coordinates (t, r). The golden spiral is a particular case of a logarithmic spiral: it is described by the equation

$r(t) = \varphi^{\frac{2}{\pi}t}$, where φ is the golden ratio. As was explained above, there is no significant evidence that shells have the golden ratio parameter built-into their growth model. The shapes of shells may be well modeled by a logarithmic spiral, but from Falbo's measurements it follows that they are not modeled by a golden spiral.

- **The golden ratio is present everywhere in the proportions of ancient Greek architecture.**

We invite the reader to browse on the web for numerous illustrations serving to check these words of M. Gardner and G. Markowsky:

M. Gardner: "The efforts of phi cultists to find golden rectangles in architecture, painting, and sculpture reached absurd heights. It is easy to see how this happened. Measurements of parts of a building, or work of art, have fuzzy boundaries such that it is easy to find phi when ratios close to phi fit just well." [**55**, p. 244].

G. Markowsky: "Many sources (...) claim that the Parthenon embodies the golden ratio in some way. To support this claim authors often include a figure where the large rectangle enclosing the end view of the Parthenon-like temple is a golden rectangle. None of these authors is bothered by the fact that parts of the Parthenon are outside the golden rectangle." [**54**, p. 8].

- **Leonardo da Vinci used the golden ratio for his paintings, and this fact adds to the perfect beauty of his artworks**.

The arguments are usually based not on any historical evidence, but on some measurements that are rather idiosyncratic, as in the case of the Parthenon, to be accepted as a serious argument. G. Markowsky mentions the possible source of this common myth:

G. Markowsky: "The claims that Leonardo da Vinci used the golden ratio seem to be based on the fact that he illustrated Luca Pacioli's book De Dvina Proportione. The biographies of Leonardo da Vinci (...) give no indication that he used the golden ratio in paintings or drawings not intended for Pacioli's book." [**54**, p. 11].

We would like to conclude this section with the words of the Italian writer and philosopher Umberto Eco that ingeniously captured the absurdity of numerology. One of the characters of his novel *Foucault's Pendulum* [**58**] performed the following elegant exercise in numerology based on the measurements of an ordinary lottery-ticket kiosk:

U. Eco: " ... I invite you to go and measure that kiosk. You will see that the length of the counter is one hundred and forty-nine centimeters - in other words, one hundred-billionth of the distance between the Earth and the Sun. The height at the rear, one hundred and seventy-six centimeters, divided by the width of the window, fifty-six centimeters, is 3.14. The height at the front is nineteen decimeters, equal, in other words, to the number of years of the Greek lunar cycle. The sum of the heights of the two front corners and the two rear corners is one hundred and ninety times two plus one hundred and seventy-six times two, which equals seven hundred and thirty-two, the date of the victory at Poitiers. The thickness of the counter is 3.10 centimeters, and the width of the cornice of the window is 8.8 centimeters. Replacing the numbers before the decimals by the corresponding letters of the alphabet, we obtain C for ten and H for eight, or $C_{10}H_8$, which is the formula for naphthalene." [**58**, p. 288]

6.7 Phyllotaxis and Examples of the Golden Ratio and Fibonacci Numbers in Modern Art

Despite the questionable statements discussed above, we would like to reassure the reader that, yet, there are amazing things about the golden ratio that are confirmed to be true. First, the golden ratio remains a very interesting number with remarkable mathematical properties. Second, there are indeed cases in nature and art, where the Fibonacci numbers or the golden ratio play a role.

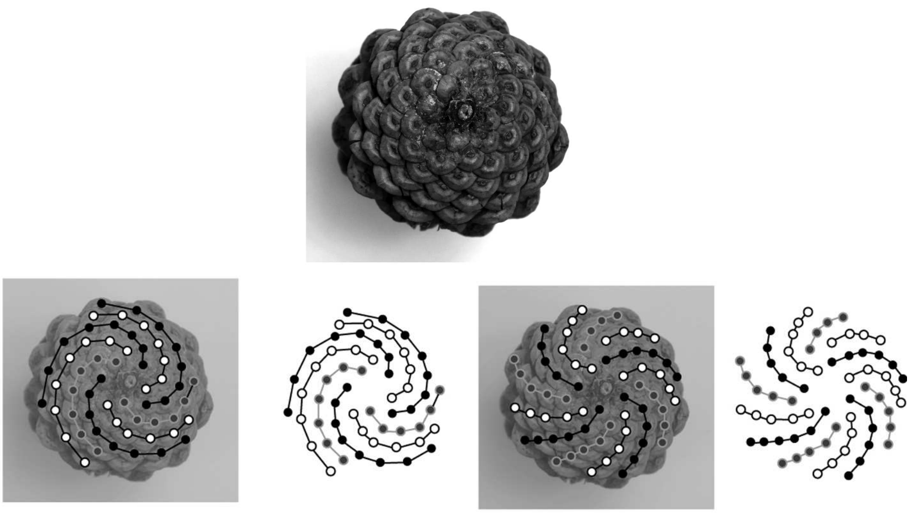


FIGURE 6.3. 8 and 13 spirals can be seen on this pinecone.

6.7.1 Pinecones This is an amazing fact: take an average pinecone and count the number of left and right spirals created by scales. Very likely, the two numbers will be consecutive Fibonacci numbers. The reader can experimentally check this phenomenon by studying a regular pinecone, a sunflower, an artichoke, or a pineapple. The evidence of this tendency is supported by the data collected from different sources in the book **[59]** by the mathematical biologist Roger V. Jean. From 12,750 observations on more than 650 species, the Fibonacci sequence $(1, 2, 3, 5, 8, ...)$ appeared in 91.3% of the cases. The sequence $(2, 4, 6, 10, 16, ...)$ was observed in 5.2% of the cases, and the sequence $(1, 3, 4, 7, ...)$ was observed in 1.5% of the cases. There were other sequences that appeared in significantly less than 1% of the cases. Note that all three above-mentioned sequences satisfy the same recurrence relation: each next number is the sum of the preceding two. Then it is natural to look for the reasons for this phenomenon. That turns out to be a hard question, and it seems that the answer is not yet understood. Phyllotaxis –the above-described phenomenon–is the subject of research in mathematical biology. We refer to the book **[59]**, which is devoted to understanding the phenomenon of phyllotaxis and is addressed to professional biologists and mathematicians (see also the references therein).

6.7.2 Modulor There are examples of modern architecture and art where the golden ratio and Fibonacci numbers were used intentionally. We know that for sure since it is well documented by the creators of this art. Charles Edouard Jeanneret (1887–1965), widely known to the world as the architect Le Corbusier, was very much fascinated by the so-called regulating lines in composition and architectural design. The idea of the golden ratio describing the perfect proportions occupied the minds of members of Le Corbusier's architecture bureau, questing to find an instrument, a template of measurements to design effortlessly buildings of harmonious proportions. Le Corbusier and his collaborators attempted to find geometrically, using mathematics, a system of measurements that would have the following properties.

- It would refer to human proportions and would resolve mismatch between the Anglo-Saxon and French metric systems.

- It would offer guidelines to constructors how to build from prefabricated materials buildings of beautiful proportions.
- The system would be based on the so-called right angle rule and the golden ratio proportion.

Le Corbusier proposed a system, called *Modulor*, which is most often represented by a diagram of a man with a raised hand. The reproduction of the Modulor in Figure 6.4 is based on one of the early versions of the diagram.

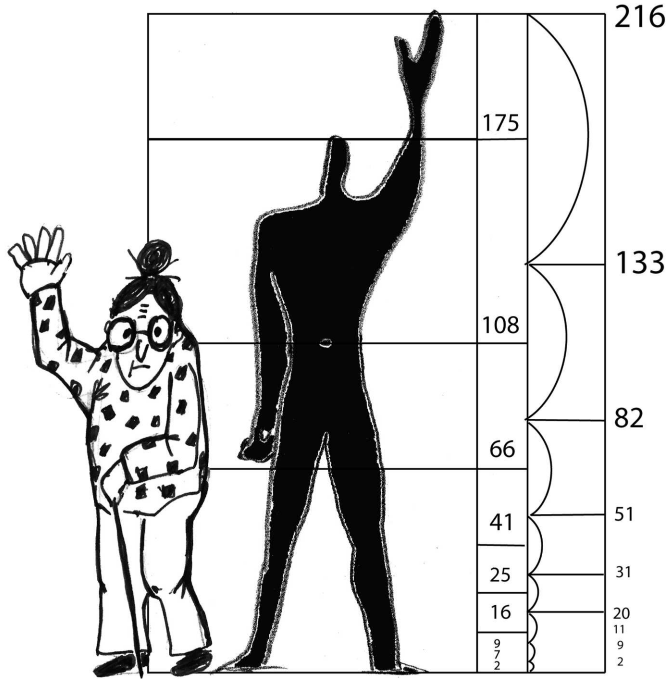


FIGURE 6.4. The diagram of Modulor Man (after Le Corbusier).

Le Corbusier was not a very strong mathematician, and his deductions contain numerous mathematical mistakes. From the point of view of mathematics, Le Corbusier has failed to solve the problem and his geometric constructions were wrong and impossible. However, in the course of experiments with proportions, Le Corbusier, literally measuring his diagrams with a tape, arrived at the following two sequences:

$$2,\quad 7,\quad 9,\quad 16,\quad 25,\quad 41,\quad 66,\quad 108,\quad 175, \tag{6.7}$$

$$2,\quad 9,\quad 11,\quad 20,\quad 31,\quad 51,\quad 82,\quad 133,\quad 216. \tag{6.8}$$

He noticed that both sequences satisfy the same rule as Fibonacci numbers: *Each next number is the sum of the preceding two.* (The discrepancy for 108, 175, and 216 is caused by the fact that Le Corbusier did not really do geometric constructions or proofs, but rather measurements with a special tape. Therefore, the experimental error of his measurements accumulated.)

It is worth mentioning that Le Corbusier was aware of the flaws of his "deductions." His younger collaborator Gérald Hanning[15] and the mathematician René Taton pointed out early that the whole construction is mathematically wrong. Nevertheless, Le Corbusier did not pay much attention to their criticism and wrote the book *Modulor* **[60]** to claim that he mathematically deduced new rules of harmony. He advertised his new system of measurements wherever it was possible. The idea of finding mathematical rules for beauty was very appealing to many architects and designers of that time, and there was a lot of interest in Le Corbusier's concepts. The architect built several buildings based on the Modulor system of proportions, the most famous one is the apartment building *Unité d'Habitation* in Marseille (France).

6.7.3 Other examples

- The Italian artist Mario Merz (1925–2003) used Fibonacci numbers in his art as the signature feature. The artist often introduced neon words and Fibonacci numbers in his installations. Mario Merz was associated to the Italian movement Arte Povera.
- Section d'Or is the name of a group of French cubist painters who worked in Paris in 1912–1914.
- Eden's educational project in Cornwall (UK) was designed by Nicholas Grimshaw and the engineering firm Anthony Hunt and Associates. The roof of the Core building and the granite Seed sculpture in the center of this building have the scales going in Fibonacci–pattern spirals.
- An interesting story of a mistake contained in the Fibonacci–numbers–based sculpture *Boundaries of Infinity* by Norbert Francis Attard in De Panne (Belgium) can be read in **[61]**.
- There is an obvious example of a famous modern building that has the golden ratio proportions associated to it since that building has the form of a regular pentagon: the building of the United States Department of Defense in Washington. Somehow, nobody

[15] At an earlier stage of the project, Gérald Hanning proposed a mathematically correct version of the diagram, but Le Corbusier favored his own faulty version.

ever mentioned it as a harmonious example of the architecture of the XXth century.

6.8 Geometric Construction of Golden Ratio

We describe the steps of constructing a golden rectangle in a classical way by using a compass and a straightedge. The Pythagorean theorem shows that the sides of the constructed rectangle satisfy the golden ratio relation.

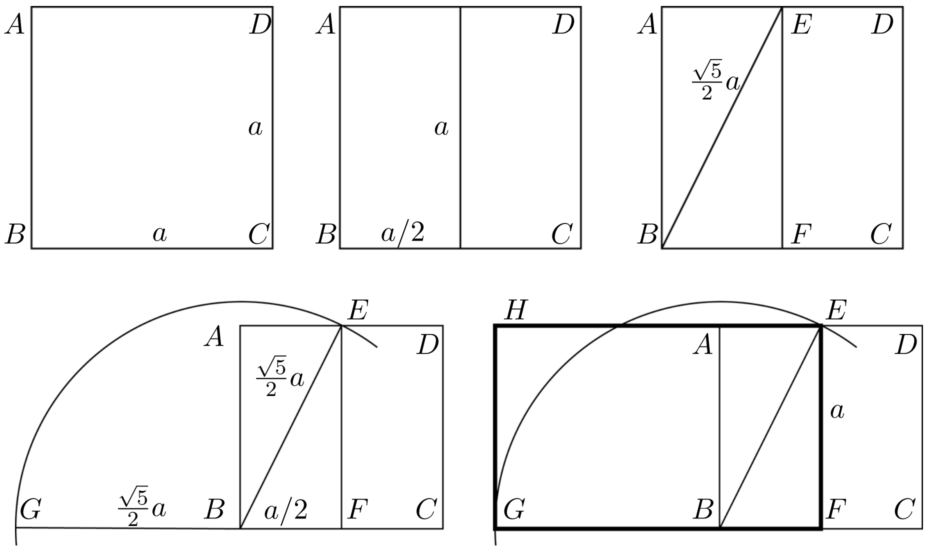


FIGURE 6.5. Construction of a golden rectangle.

***Step* 1** Draw a square $ABCD$ of side length a.

***Step* 2** Connect the midpoints of the opposite sides of the square. The square is now divided into two rectangles of size $a/2 \times a$.

***Step* 3** Draw a diagonal in one of these rectangles (the segment BE in the third diagram). By the Pythagorean theorem, BE has the length

$$\sqrt{a^2 + a^2/4} = a\frac{\sqrt{5}}{2} \text{ units.}$$

***Step* 4** Using a compass, construct the segment BG of length $a\sqrt{5}/2$ as in the fourth diagram.

***Step* 5** The segment FG has length $a(\sqrt{5}+1)/2$, and the segment FE has length a. Thus, the ratio of the sides of the rectangle $HEFG$ is $(\sqrt{5}+1)/2$. It is a golden rectangle.

It is also well known that the diagonals of a regular pentagon are related to its sides in the proportion of the golden ratio. The construction of regular pentagon can be found, for example, in [**62**]. The following two problems from [**55**] describe less-known ways to see geometrically the golden ratio.

Problem 6.3. The triangles in Figure 6.6 are equilateral. The side length of each of the three small white triangles is a, and the side length of each of the four small grey triangles is b. Prove that b/a is the golden ratio.

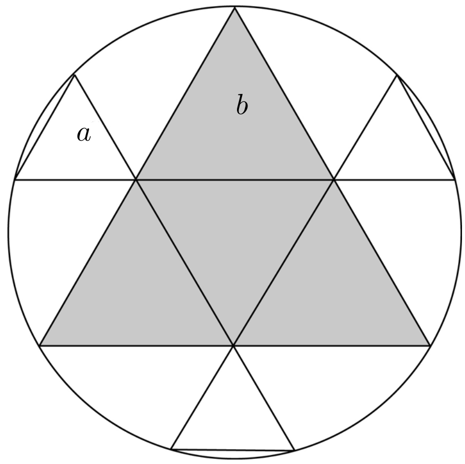


FIGURE 6.6.

Problem 6.4. Prove that, in Figure 6.7, the radius of the large semicircle divided by the diameter of one of the three identical grey circles is the golden ratio.

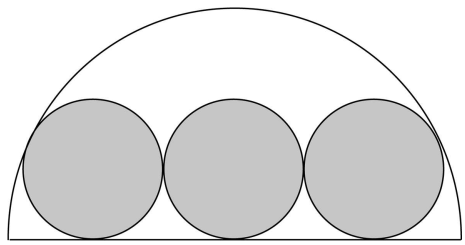
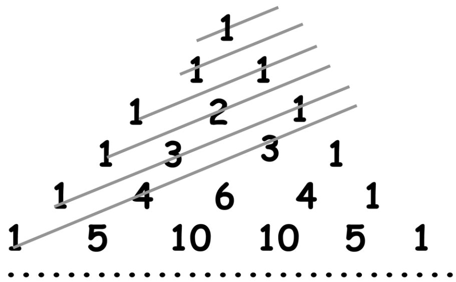


FIGURE 6.7.

6.9 Problems on Properties of Fibonacci Numbers

Problem 6.5. Mary grows fruits in her garden. Every day she either works in her garden or goes to town to sell her fruits, a two day round trip. A round trip to town takes exactly two days. A schedule is complete if all the trips are complete. For example, during three consecutive days, Mary's complete schedule may look like (G, G, G), or $(2T, G)$, or $(G, 2T)$. Here, G stands for gardening and $2T$ means two days for the round trip to town. During four consecutive days, her complete schedule may look like (G, G, G, G), or $(G, G, 2T)$, or $(G, 2T, G)$, or $(2T, G, G)$, or $(2T, 2T)$. Prove that for n consecutive days $F(n+1)$ different schedules are possible, where $F(n+1)$ is the $(n+1)^{\text{st}}$ Fibonacci number.

Problem 6.6. Add the sums along the shallow diagonals of Pascal's triangle.



Using Problem 6.5, show that each of these sums is a Fibonacci number.

Problem 6.7. Prove that $F(1) + F(2) + \cdots + F(n) = F(n+2) - 1$.

Problem 6.8. Prove that $F(1)^2 + F(2)^2 + \cdots + F(n)^2 = F(n)F(n+1)$.

Problem 6.9. For what values of n are the Fibonacci numbers $F(n)$ even?

Problem 6.10. Prove that $F(n+m) = F(n-1)F(m) + F(n)F(m+1)$ for any n and m.

Problem 6.11. Using the result of Problem 6.10, prove that $F(200)$ is divisible by $F(4) = 3$.

Problem 6.12. Let

$$f(x) = \sum_{n=0}^{\infty} F(n)x^n$$

be the generating function for the Fibonacci numbers. Prove that for $|x| < (1+\sqrt{5})/2$ the series $f(x)$ is convergent and

$$f(x) = \frac{x}{1-x-x^2}. \tag{6.9}$$

6.10 Playing with Sequences

The pattern of the table unwinds in the form of a spiral. Actually, any sequence of positive real numbers can be used to draw patterns in the plane (though examples below show that they do not always look like spirals). Suppose that we have a sequence of numbers $a(1)$, $a(2)$, $a(3)$, $a(4), \ldots$. Take a piece of graph paper. Mark the starting point and draw a straight segment of length $a(1)$. Make the right 90° turn and draw a straight segment of length $a(2)$ in the new direction. Again make the right 90°, turn, and draw a straight segment of length $a(3)$, and so on. For example, the Fibonacci sequence $1, 1, 2, 3, 5, 8, \ldots$ gives the spiral D in Figure 6.8.

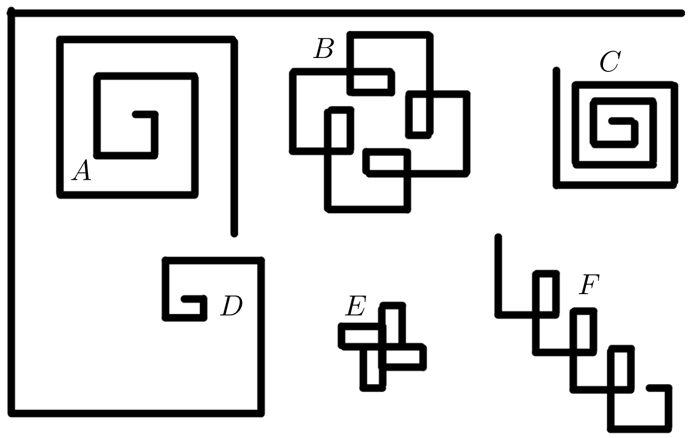


FIGURE 6.8.

Problem 6.13. Match the sequences with the patterns in Figure 6.8.

(a) 1, 2, 3, 4, 5, 6, 7, ...
(b) 1, 1, 2, 2, 3, 3, 4, 4, 5, 5,...
(c) 1, 2, 3, 1, 2, 3, 1, 2, 3,...
(d) 1, 2, 3, 4, 1, 2, 3, 4,...
(e) 1, 2, 3, 4, 5, 1, 2, 3, 4, 5,...

Chapter 8

LINEAR PERSPECTIVE

FIGURE 8.1. *No Masters* by Carol Pylant.
KSU, Beach Museum of Art.

8.1 Space Presentation

Landscapes, interiors, portraits... Artists are frequently challenged by the task of depicting space and relative positions of objects: the world around us has three dimensions, but the canvas is flat. Some artists search for abstract solutions to this problem, and others carefully reproduce space in a realistic manner. How do the latter show the depth of space on a flat surface? One can try to trick the eye by the same clues which help to estimate depth and distance in the real world. In particular, we know that

- objects that are farther away look smaller,

- objects in front can obscure the view of parts of objects that are in the back.

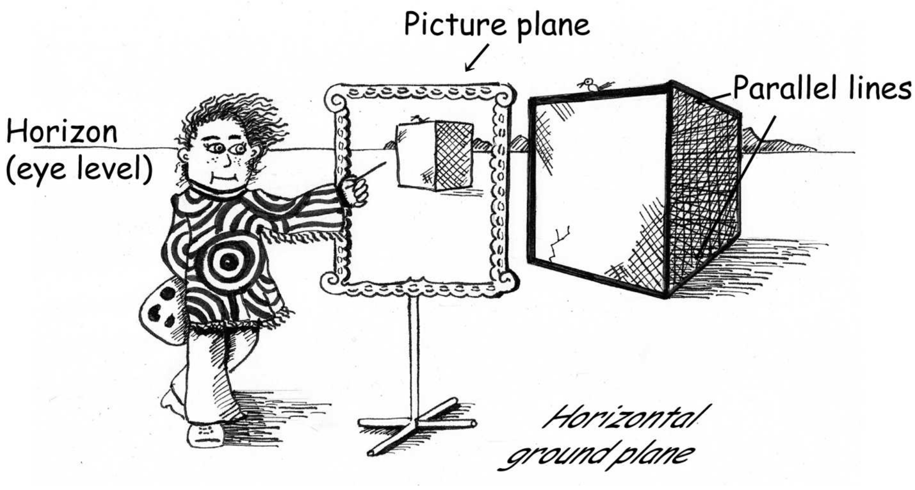
Linear perspective is a geometry based method that allows artists to reproduce such clues in a consistent way. Application of linear perspective helps the viewer to judge from a two-dimensional image about relative proportions of the objects and their relative placement in three-dimensional space.

The method of linear perspective is always at the top of the list of connections between art and geometry. Hundreds of books explain the fundamentals of this method for all types of readers: artists, designers, architects, computer scientists, math educators, historians, young readers, etc. The knowledge of basic principles of the method is handy not only to those who create images, but also to those who view the results of artists' work. Understanding linear perspective leads to better understanding of art. For example, look at the painting *No Masters* by Carol Pylant (see Figure 8.1). Can you answer the following questions:

- Where is a horizon line in the picture?
- Is the viewer of the scene *No Masters* standing or sitting?
- Were the rules of geometric perspective followed rigorously?

Artworks from the collection of the Marianna Kistler Beach Museum of Art will help us to review the main ideas of linear perspective.

8.2 Main Features of Linear Perspective

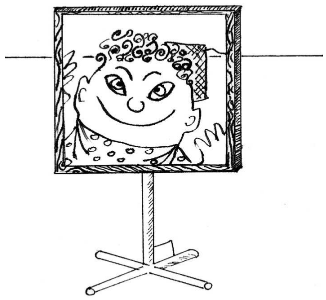


Key components of a linear perspective composition.

A standard linear perspective drawing has the following key components:

- the picture plane
- the horizontal ground plane
- the eye level of the observer or the level of the camera
- groups of parallel lines that define the shapes of the objects in the picture (edges of roads, edges of facades of buildings, etc.)

8.3 Picture Plane and Ground Plane



The picture plane (canvas) is a flat two-dimensional plane, where the image is to be created. We assume that artist's intention is to draw the scene as realistically as possible. Then the composition is determined by positions of objects relative to the picture plane and to the artist. In what follows, the *ground plane* means a horizontal flat surface where the artist with his canvas or camera is positioned. It can be level ground or a floor of a room. The angle between the picture plane and the ground plane is assumed to be 90°. Of course, there may be a situation where the artist is standing on a slope of a hill, so the angle is not 90° and the ground plane is not horizontal (see Figure 8.2). For the sake of simplicity we do not consider such cases. With appropriate adjustments, the general rules of linear perspective can be applied to such cases too, but we formulate statements below under the assumption that the main ground plane is horizontal.

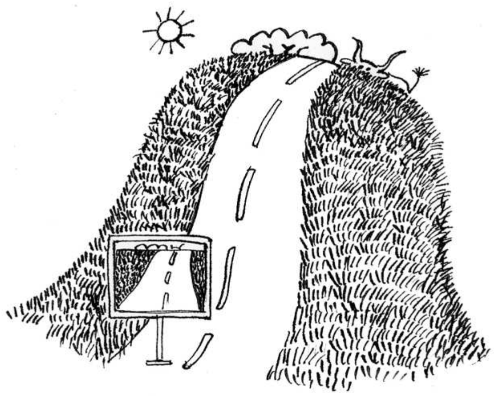


FIGURE 8.2.

summer, starting from 1982, Smith began to return to Kansas, where she stayed with her brother for a few weeks. Upon his death she bought a small trailer that she moved onto a farm not far from Whitewater. She painted the landscapes and farm animals. The paintings were later exhibited at the American Academy and Institute of Arts and Letters in New York.

The painting *Kansas Gate Posts* was displayed in the office of U.S. Senator Nancy Landon Kassebaum Baker. The Senator donated the work to the Marianna Kistler Beach Museum of Art in 1998 [**89**].

8.10 Problems on Linear Perspective

Problem 8.1. Determine the eye level of the observer of *Abandoned* by Mary Eck Holland Call (see Figure 8.17). Is the observer standing or sitting?

FIGURE 8.17. *Abandoned* by Mary Eck Holland Call.
KSU, Beach Museum of Art.

Problem 8.2. Find the approximate horizon level of the artwork *Autumn Road* by Norma Bassett Hall (see Figure 8.18).

FIGURE 8.18. *Autumn Road* by Norma Bassett Hall.
KSU, Beach Museum of Art.

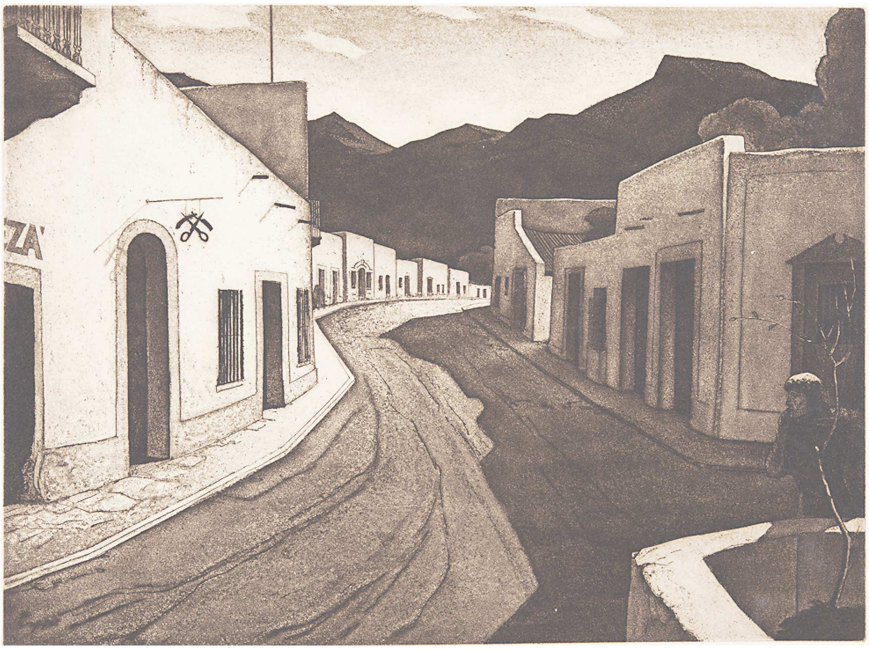


FIGURE 8.19. *Mexican Barber Shop* by Charles Merrick Capps.
KSU, Beach Museum of Art.

✵✵ **Problem 8.3.** Questions on *Mexican Barber Shop* by Charles Merrick Capps (see Figure 8.19).

(a) How many vanishing points does the perspective of the artwork use?

(b) Find the horizon level of the composition.

✵✵ **Problem 8.4.** Additional questions on the *Anderson Hall* lithograph (see Figure 8.14).

(a) Assume that the level A in Figure 8.14 is the true horizon line. What can you say about the group of people walking along the curb?

(b) Figure 8.20 is a photograph of the building taken from approximately the same position as the view of the *Anderson Hall* lithograph. How high above the ground was the camera placed to take this picture?

(c) Compare the photograph with the lithograph. What is the same, and what is different?

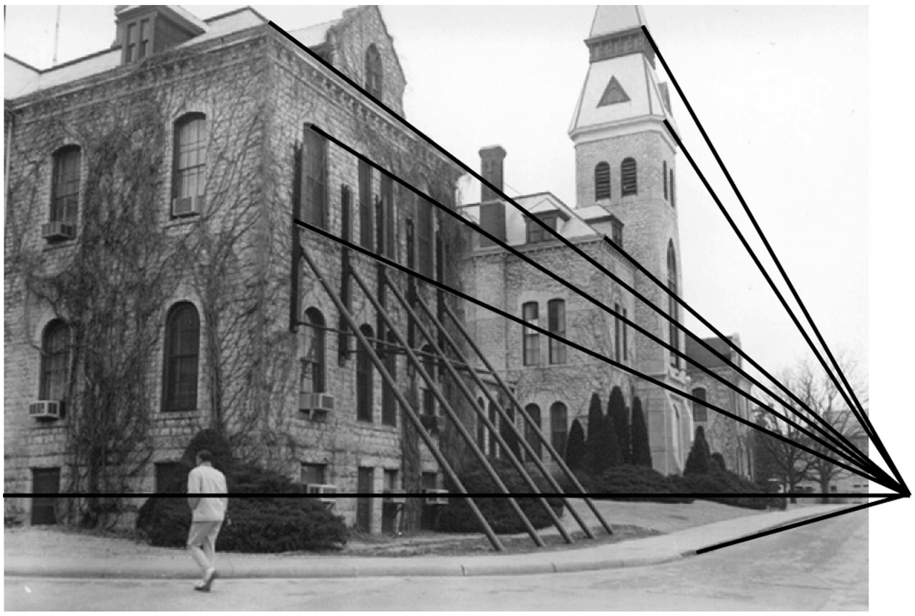


FIGURE 8.20. *Anderson Hall.* Photograph.
KSU, University Archives and Manuscripts, Richard L. D. and Marjorie J. Morse Department of Special Collections.

✵ **Problem 8.5.** Additional question on *Spring and Mercer* by Carol Pylant (see Figure 8.12). The girl stands farther away than the woman, so she is depicted smaller in the painting. Nevertheless, it is possible to compare the heights of the girl and the woman. How?

Problem 8.6. Questions on the *No Masters* painting by Carol Pylant (see Figure 8.1).

(a) How many vanishing points does the composition use?

(b) Find the horizon line of the composition. Recall that this is an eye level of an observer of the scene. Is the viewer standing or sitting?

(c) Did you find any deviations from rigorous rules of linear perspective?

Problem 8.7. Look at Figure 8.21. Do you note anything strange?

FIGURE 8.21. Illustration: Strange Space.

Problem 8.8. Artists not only use linear perspective to create an illusion of a three-dimensional space, but they sometimes play with it. Recall the famous example of the painting *The Ambassadors* by Hans Holbein the Younger in the collection of the London National Gallery (UK). In the foreground of the painting, at the feet of the noblemen there is a distorted shape. One has to look at the painting from the right side under an angle to recognize in that shape a skull.

(a) Look at Figure 8.22. Can you read what is written there? Is it easier to read it if you place the book horizontally in front of your eyes?

(b) Make your own distorted image. The distortion should be corrected when the viewer looks at your picture under a certain angle.

REFERENCES

Chapter 1

1. Geographic Center of the United States of America Lebanon, Kansas.
   www.kansastravel.org/geographicalcenter.htm
2. Results of calculation by Daft Logic Distance Calculator.
   www.daftlogic.com/projects-google-maps-distance-calculator.htm
3. U.S. Census Bureau 2010.
   www.ci.manhattan.ks.us/DocumentCenter/Home/View/9663
4. K-State Media Guide – Enrollment numbers.
   www.k-state.edu/media/mediaguide/enrollment.html
5. Riley County Historical Museum.
   http://rileycountyhistoricalmuseum.weebly.com
6. W. G. Cutler, *History of the State of Kansas*, Riley County, Part 6: Manhattan, Part 2, A. T. Andreas, Chicago, IL (1883).
   www.kancoll.org/books/cutler/riley
7. W. N. Slagg, *Riley County, Kansas: A Story of Early Settlements, Rich Valleys, Azure Skies and Sunflowers*, Theo. Gaus' Sons, Brooklyn, NY (1968).
8. Kansas Historical Society. www.kshs.org
9. Kansapedia. www.kshs.org

Chapter 2

10. *Alan Shields: A Survey*, the Beach Museum of Art & Kansas State University, Manhattan, KS (1999).
11. D. Emont Scott, *Alan Shields*, Memphis Brooks Museum of Art, Memphis, TN (1983).
12. Ch. Masters, "Alan Shields, American artist known for his stitching, sewing, collages and use of grid systems," *The Guardian*, February 28, 2006.
    www.theguardian.com/news/2006/feb/28/guardianobituaries.usa
13. B. North, *...to Build up a Rich Collection. Selected Works from the Marianna Kistler Beach Museum of Art*, the Marianna Kistler Beach Museum of Art and Kansas State University, Manhattan, KS (2003).
14. H. S. M. Coxeter, *Regular Polytopes*, 3rd ed., Dover (1973).
15. H. S. M. Coxeter, *Introduction to Geometry*, 2nd ed., Wiley (1989).
16. *Geometry – Making a Paper Plate Icosahedron.*
    http://learningideasgradesk-8.blogspot.com/2011/11/ geometry-making-paper-plate-icosahedron.html

Chapter 3

17. F. N. David, *Games, Gods and Gambling. A History of Probability and Statistical Ideas*, Dover (1998).
18. K. J. Devlin, *The Unfinished Game. Pascal, Fermat, and the Seventeenth-Century Letter that Made the World Modern. A Tale of How Mathematics is Really Done*, Basic Books, New York (2008).
19. See [**13**].
20. *Unique Creative Visions: The Art of Jessie Montes.* Flyer, Duane and Orvileta West.
21. *3 Standard Stoppages by Marcel Duchamp, Paris 1913-14*, The Museum of Modern Art (US).

    www.moma.org/collection/works/78990?locale=en
22. G. R. Grimmett and D. R. Stirzaker, *Probability and Random Processes*, 3rd ed., Oxford University Press, Oxford (2001).

Chapter 4

23. J. H. Conway, H. Burgiel, and Ch. Goodman-Strauss, *The Symmetries of Things*, A K Peter, Wellesley, MA (2008).
24. *Jack Chevalier.* Linda Hodges Gallery

    www.lindahodgesgallery.com/#!jack-bio/c1wf3
25. S. Peterson, *Lucy M. Lewis: American Indian Potter*, 2nd ed., Kodansha (2004).
26. M. Ellen and L. R. Blair, *Margaret Tafoya. A Tewa Potter's Heritage and Legacy*, Shiffer Publishihng Ltd., West Chester, Pennsylvania (1986).
27. *Seven Families in Pueblo Pottery. Maxwell Museum of Anthropology* (Reprint edition), University of New Mexico Press, Albuquerque (1975).
28. R. Dillingham, *Fourteen Families in Pueblo Pottery 1994*, University of New Mexico Press (1994).
29. L. D. Holmes, *Through the Eyes of the Pot*, A Study of Southwest Pueblo Pottery and Culture, Museum of Antropology, Wichita State University.

    www.holmes.anthropology.museum/southwestpottery/map.html
30. S. Trimble, *Talking with the Clay. The Art of Pueblo Pottery*, School of American Research Press, Santa Fe, New Mexico (1987).
31. K. M. Chapman, *The Pottery of San Ildefonso Pueblo*, The School of American Research University of New Mexico Press, Albuquerque (1970).
32. F. H. Harlow, *Modern Pueblo Pottery 1880-1960*, Northland Press, Flagstaff (1977).
33. Bruce Bernstein and J. J. Brody, *Voices in Clay. Pueblo Pottery from the Edna M. Kelly Collection*, Miami University Art Museum, Oxford, Ohio (2001).
34. D. K. Washburn, *A Symmetry Analysis of Upper Gila Area Ceramic Design*, Peabody Museum of Archeology and Ethnology, Harvard University, Cambridge, Mass. (1977).

Chapter 5

35. Cicero, *De Natura Deorum Academica, Vol. XIX* English transl. by H. Rackham, Harvard Univ. Press, Cambridge, and William Heinemann Ltd., London (1967).

36. "University of New Hampshire and the New Hampshire College of Agriculture and the Mechanic Arts. Catalog 1942–1943: The University Faculty and Staffs," *Bull. Univ. New Hampshire* **33**, No. 8, p. 26 (1941).

www.archive.org/details/universityofn19421943univ

37. "The Portsmouth Herald from Portsmouth," *The New Hampshire*[25], October 16, 1941.

38. *Parsons.* website: www.newschool.edu/parsons/history

39. *Sculptured Painting, A New Visual Experience in Dimensional Expression*, Flyer of the Exhibition at Saint John's Hospital, Santa Monica, California.

40. Archives of the Marianna Kistler Beach Museum of Art, Manhattan, KS.

41. *Kisilev's Geometry. Book I. Planimetry. Book II. Stereometry.* Adapted from Russian by Alexander Givental, Sumizdat (2016).

42. V. I. Arnold, *Problems for Children from 5 to 15* [in Russian], Moscow Center for Continuous Mathematical Education, Moscow (2004).

43. V. Gutenmacher and N. B. Vasilyev, *Lines and Curves: A Practical Geometry Handbook*, Birkhäuser, Basel (2004).

44. L. Hall and S. Wagon, "Roads and wheels," *Math. Mag.* **65**, No. 5, 283–301 (1992).

45. E. R. Russell, G. Luttrell, and M. Ric, "Roundabout studies in Kansas," In: *Proceedings of the 30 th CSCE Annual Conference, Montreal, Quebec, Canada* **5**, pp. 2283-2292 (2002).

http://transport.ksu.edu/center/research

http://toc.proceedings.com/01701webtoc.pdf

Chapter 6

46. Ch. Stroh, *Kansas State University 1980-1990: Department of Art, KSU Friends of Art, the Marianna Kistler Beach Museum of Art*, Fidlar Doubleday, Davenport, IA (2012).

47. R. A. Brualdi, *Introductory Combinatorics*, 5th ed., Pearson (2009).

48. R. L. Graham, D. E. Knuth, and O. Patashnik, *Concrete Mathematics: A Foundation for Computer Science*, 2nd ed., Addison-Wesley Publ. Co., Amsterdam (1994).

49. *The Mercury* [26], November 1, 2009.

50. *The Office of University News*, September 25, 1963. [Archives of Kansas State University].

51. *The Collegian* [27], May 19, 1964.

[25] The University of New Hampshire's student newspaper since 1911.
[26] The local newspaper for Manhattan, Kansas.
[27] The student newspaper of Kansas State University in Manhattan.

52. *The Collegian*, October 26, 1983.
53. L. Komer, "K-State sculptures' histories revealed," *K-State Collegian*, October 7, 2013.
54. G. Markowsky, "Misconceptions about the golden ratio," *College Math. J.* **23**, No. 2, 1–19 (1992).
55. M. Gardner, "Cult of the golden ratio," *Skept. Inq.* **18**, 243–247 (1994).
56. C. Falbo "The golden ratio – A contrary viewpoint," *College Math. J.* **36**, No. 2, 123–134 (2005).
57. R. Herz-Fischer, "The home of golden numberism," *Math. Intell.* **27**, No. 1, 69–71 (2005).
58. U. Eco. *Foucault's Pendulum*, Book Harcourt Brace Janovich, Publishers (1989).
59. R. V. Jean, *Phyllotaxis: A Systematic Study in Plant Morphogenesis*, Cambridge University Press, Cambrdige (1994).
60. Le Corbusier, *Le Modulor*, Birkhäuser, Basel (2000).
61. D. Huylebrouck, "The skew Fibonacci spiral of Belgium," *Math. Intell.* **36**, No. 4, 74–77 (2014).
62. R. Hartshorne, *Geometry: Euclid and Beyond*, Springer, Berlin (2000).
63. *The On-Line Encyclopedia of Integer Sequences.* https://oeis.org
64. D. S. Gunderson, *Handbook of Mathematical Induction: Theory and Applications*, CRC Press, Boca Raton, FL (2011).

Chapter 7

65. *The Kansas Landscape*, Kansas Department of Economic Development, Topeka, KS (1985).
66. C. E. Swartz, *Phenomenal Physics*, John Wiley and Sons, New York etc. (1981).
67. *2009-2010 K-State University Faculty Catalogue*, Kansas State University, Manhattan, KS (2009).
68. See [**46**].
69. *1981 KSU Art Newsletter*, Kansas State University, Manhattan, KS (1981).

Chapter 8

70. *The Elements of Art: Using perspective to represent space* [educational video at the Louvre website]:

    www.louvre.fr/en/clefanalyses/using-perspective-represent-space
71. M. Wagner, *The Geometries of Visual Space*, Psychology Press (2006).
72. Harold L. Doolittle, "The Printmakers Society of California sends greetings to its associate members," Leaflet of November 1951.
73. Donald N. Brown, "Charles M. "Chili" Capps 1898 - 1981," Essay for an exhibition catalog (2007).

    http://casewardprintmaker.com/Mentor/Chili_Capps.html
74. Classic Covers from the Post / Artists / Stevan Dohanos at *The Saturday Evening Post.* www.saturdayeveningpost.com

75. "Stevan Dohanos, A Stamp Designer And Illustrator, 87." Obituary, *The New York Times*, July 6, 1994.

www.nytimes.com/1994/07/06/obituaries/stevan-dohanos-a-stamp-designer-and-illustrator-87.html

76. J. Patterson, *Norma Bassett Hall: Catalogue Raisonne of the Block Prints and Serigraphs*, Pomegranate Communications, Portland, Oregon (2014).

77. *Norma Bassett Hall 1889 - 1957.*

http://casewardprintmaker.com/Mentor/Norma_Bassett_Hall.html

78. Archive of the Marianna Kistler Beach Museum of Art. Past Exhibitions 2011-2013.

79. *George M. Kren (1926-2000).* Obituary. *Clio's Psyche.*

www.cliospsyche.org/obituaries_gk.html

80. *Roy Langford Paintings.* An online gallery of paintings by Roy Langford of Manhattan, Kansas. website: http://roylangford.blogspot.com

81. *Roy Langford.* The Strecker-Nelson Gallery.

www.strecker-nelsongallery.com/roy-langford-bio

82. *Carol S. Pylant. Artist. Emeritus Professor, University of Wisconsin-Madison.* website: www.carolpylant.com

83. *Carol Pylant Profile (Art - Painting).*

www.youtube.com/watch?v=2AbfjiGk0CU

84. L. James, *Standing Stones*, Essay (1998). www.carolpylant.com/Essay_1998.html

85. *An interview with Larry Schwarm, winner of the 2002 CDS/Honickman First Book Prize in Photography.*

https://vimeo.com/28431682

86. *Larry Schwarm. Photography.* website: www.larryschwarm.com

87. *K-Stater*, [28] October, 1999.

88. *Shirley Smith.* website: www.shirleysmithnyc.com/

89. "Memorial service for actress, Kansas native Shirley Smith, 84" *The Wichita Eagle*, December 20, 2013.

www.kansas.com/news/article1130142.html

Chapter 9

90. See [**48**].

91. G. A. Jones and J. M. Jones, *Elementary Number Theory*, Springer, Berlin (1998).

92. *The Rückl & Sons Glassworks Factory.* website: www.ruckl.cz

Chapter 10

93. Mitsugi Ohno, *A Japanese in Kansas: A Man Captivated by Glass*, Tsutomu Ohno (2006).

[28] The magazine published by K-State Alumni Association.

94. “Ohno ships set sail to Union,” *K-State Student Union News*, May 7, 2015.
95. V. G. Boltyanskii and V. A. Efremovich, *Intuitive Combinatorial Topology*, Springer, New York (2001).
96. V. V. Prasolov, *Intuitive Topology*, Am. Math. Soc., Providence, RI (1995).
97. I. Moscovich, *The Big Book of Brain Games: 1,000 Play Thinks of Art*, Workman Publish. Co. (2006).

ARTWORK INDEX

1. *Abandoned* (1950) by Mary Eck Holland Call
KSU, Beach Museum of Art, Manhattan, Kansas (USA)
Math topic: Linear perspective. Horizon (Chapter 8, p. 148)

2. *Achilles and Ajax Engaged in a Game* (ca. 540-530 BC). Amphora by Exekias
Vatican Museums (Vatican City)
Math topic: Combinatorics and probability. Games of chance
(Chapter 3, p. 45)

3. *Anderson Hall* (1947) by Charles Merrick Capps
KSU, Beach Museum of Art, Manhattan, Kansas (USA)
Math topic: One-point perspective (Chapter 8, p. 137)

4. Artworks by Maurits Cornelis Escher
Math topic: Polyhedra (Chapter 2, p. 28)

5. *Autumn Road* (ca. 1928) by Norma Bassett Hall
KSU, Beach Museum of Art, Manhattan, Kansas (USA)
Math topic: Linear perspective. Vanishing points (Chapter 8, p. 149)

6. *Biosphère* designed by Richard Buckminster Fuller
Montreal (Canada)
Math topic: Polyhedra (Chapter 2, p. 29)

7. Blackware jar (20th century) by Naomi Gutierrez
KSU, Beach Museum of Art, Manhattan, Kansas (USA)
Math topic: Symmetry. Frieze pattern (Chapter 4, p. 65)

8. Bowl. Glazed earthenware (20th century) by Maria Margarita Tafoya
KSU, Beach Museum of Art, Manhattan, Kansas (USA)
Math topic: Symmetry. Frieze pattern (Chapter 4, p. 64)

9. *Boundaries of Infinity* by Norbert Francis Attard
(Belgium)
Math topic: Fibonacci numbers (Chapter 6, p. 100)

10. *Card Players in an Opulent Interior* (ca. 1663–1665) by Pieter de Hooch
The Louvre Museum, Paris (France)
Math topic: Combinatorics and probability. Games of chance
(Chapter 3, p. 45)

11. *Collage Arranged According to the Laws of Chance* (1916) by Jean Arp
The Museum of Modern Art, New York (USA)
Math topic: Combinatorics and probability. Randomness (Chapter 3, p. 46)

12. *Color Studies: Squares and Concentric Circles* (1913) by Wassily Kandinsky
The Lenbachhaus, Munich (Germany)
Math topic: Circles (Chapter 5, p. 77)

13. *Dice, Packet of Cigarettes and Visiting Card* (1914) by Pablo Picasso
Yale University (USA)
Math topic: Combinatorics and probability. Games of chance
(Chapter 3, p. 46)

14. *Dymaxion Map* by Richard Buckminster Fuller
Montreal (Canada)
Math topic: Polyhedra (Chapter 2, p. 29)

15. Eden's educational project in Cornwall (UK) designed by Nicholas Grimshaw
Math topic: Fibonacci numbers (Chapter 6, p. 100)

16. Glazed earthenware by Orval F. Hempler
KSU, Beach Museum of Art, Manhattan, Kansas (USA)
Math topic: Reflection symmetry (Chapter 4, p. 62)

17. Glazed earthenware by Orval F. Hempler
KSU, Beach Museum of Art, Manhattan, Kansas (USA)
Math topic: Circles (Chapter 5, p. 75)

18. Glazed earthenware by Orval F. Hempler
KSU, Beach Museum of Art, Manhattan, Kansas (USA)
Math topic: Topology. Tracing design (Chapter 10, p. 174)

19. *Girl Playing Knucklebones* (5th–4th centuries BC)
The Louvre Museum, Paris (France)
Math topic: Combinatorics and probability. Games of chance (Chapter 3, p. 46)

20. *Horseman* (1946) by Maurits Cornelis Escher
Math topic: Topology (Chapter 10, p. 169)

21. *Hose Co #4* (1933) by Stevan Dohanos
KSU, Beach Museum of Art, Manhattan, Kansas (USA),
Art © Estate of Stevan Dohanos/Licensed by VAGA, New York, NY
Math topic: Linear perspective. Front view (Chapter 8, p. 136; insert, pp. 144-145)

22. *Intarsia polyhedra* (1505) by Giovanni da Verona
Verona (Italy)
Math topic: Polyhedra (Chapter 2, p. 28)

23. Illustrations by Leonardo da Vinci for Luca Pacioli's book
On the Divine Proportion (1509)
Math topic: Polyhedra (Chapter 2, p. 28)

24. Installations by Mario Merz
Math topic: Fibonacci numbers (Chapter 6, p. 100)

25. *Kansas Gate Posts* (1985) by Shirley Smith
KSU, Beach Museum of Art, Manhattan, Kansas (USA)
Math topic: Linear perspective. Front view (Chapter 8, p. 131; insert, pp. 128-129)

26. *Kansas Meatball* (1985–1998) by Alan Shields
KSU, Beach Museum of Art, Manhattan, Kansas (USA)
Math topic: Polyhedra. Icosachedron (Chapter 2, p. 25)

27. *Klein Bottle* by Mitsugi Ohno
KSU, Beach Museum of Art, Manhattan, Kansas (USA)
Math topic: Topology (Chapter 10, p. 165; insert, pp. 176-177)

28. Konza Prism (1984) by Roger Lane Routson
KSU, Beach Museum of Art, Manhattan, Kansas (USA)
Math topic: Geometrical Optics. Refraction (Chapter 7, p. 109)

29. *Knucklebones* (ca. 1550–1069 BC). Egyptian Antiquities.
The Louvre Museum, Paris (France)
Math topic: Combinatorics and probability. Games of chance (Chapter 3, p. 46)

30. Lead crystal artworks by Antonín Rückl and Sons, Ltd.,
KSU, Beach Museum of Art, Manhattan, Kansas (USA)
Math topic: Symmetry. Frieze pattern (Chapter 4, pp. 66-67).
Star polygons (Chapter 9, p. 195)

31. *Melencolia I* (1514) by Albrecht Dürer
The Metropolitan Museum of Art, New York (USA)
Math topic: Polyhedra (Chapter 2, p. 28)

32. *Mexican Barber Shop* (1938) by Charles Merrick Capps
KSU, Beach Museum of Art, Manhattan, Kansas (USA)
Math topic: Linear perspective (Chapter 8, p. 149; insert, pp. 128-129)

33. *Möbius Strip I* (1961) by Maurits Cornelis Escher
Math topic: Topology (Chapter 10, p. 169)

34. *Möbius Strip II* (1963) by Maurits Cornelis Escher
The National Gallery of Canada, Ottawa (Canada)
Math topic: Topology (Chapter 10, p. 169)

35. *No Masters* (1986) by Carol Pylant
KSU, Beach Museum of Art, Manhattan, Kansas (USA)
Math topic: Linear perspective (Chapter 8, p. 138)

36. Plate. Glazed earthenware (20th century) by Delores Lewis Garcia
KSU, Beach Museum of Art, Manhattan, Kansas (USA)
Math topic: Rotational symmetry (Chapter 4, p. 63)

37. Plate, Glazed earthenware (20th century) by Tony Da
KSU, Beach Museum of Art, Manhattan, Kansas (USA)
Math topic: Symmetry (Chapter 4, p. 63)

38. *Portrait of Luca Pacioli* (ca. 1500) attributed to Jacopo de Barbari
The Capodimonte Museum, Naples (Italy)
Math topic: Polyhedra (Chapter 2, p. 28)

39. *Pyrrho Techniko* (1989) by John L. Vogt
KSU, Beach Museum of Art, Manhattan, Kansas (USA)
Math topic: Fibonacci numbers (Chapter 6, p. 89; insert, pp. 96-97)

40. *Random Distribution of 40,000 Squares Using the Odd and Even Numbers of a Telephone Directory* (1960) by François Morellet
The Museum of Modern Art, New York (USA)
Math topic: Combinatorics and probability. Randomness (Chapter 3, p. 46)

41. *Red Earth near Salamanca, Spain, May 2000* (2000) by Larry W. Schwarm
KSU, Beach Museum of Art, Manhattan, Kansas (USA)
Math topic: Linear perspective. Horizon (Chapter 8, p. 135)

42. *Red Groom* (1979) by Jack Chevalier
The Museum of Modern Art, New York (USA)
KSU, Beach Museum of Art, Manhattan, Kansas (USA)
Math topic: Reflection symmetry (Chapter 4, p. 61; insert, pp. 16-17)

43. *Roman Statue of a Girl Playing Astragaloi* (130–150 BC)
Altes Museum, Berlin (Germany).
Math topic: Combinatorics and probability. Games of chance (Chapter 3, p. 46)

44. Seed pot. Glazed earthenware (20th century) by Shirley Duran
KSU, Beach Museum of Art, Manhattan, Kansas (USA)
Math topic: Symmetry. Frieze pattern (Chapter 4, p. 65)

45. Seed pot. Glazed earthenware (20th century) by Florence Yepa
KSU, Beach Museum of Art, Manhattan, Kansas (USA)
Math topic: Symmetry. Frieze pattern (Chapter 4, p. 66)

46. *Spring and Mercer* (1983-1984) by Carol Pylant
KSU, Beach Museum of Art, Manhattan, Kansas (USA)
Math topic: Two-point perspective (Chapter 8, p. 138)

47. *The Ambassadors* by Hans Holbein the Younger
The London National Gallery (London, UK)
Math topic: Linear perspective (Chapter 8, p. 151)

48. *The Card Players* (ca. 1890–1892) by Paul Cézanne
Metropolitan Museum of Art, New York (USA)
Math topic: Combinatorics and probability. Games of chance (Chapter 3, p. 46)

49. *The Cardsharps* (ca. 1594) by Michelangelo Merisi da Caravaggio
Kimbell Art Museum, Fort Worth, Texas (USA)
Math topic: Combinatorics and probability. Games of chance (Chapter 3, p. 45)

50. The cover for the disc *The Dark Side of the Moon* of the English rock group Pink Floyd designed by by George Hardie and the designing group Hipgnosis
Math topic: Refraction of light in a prism (Chapter 7, p. 113)

51. *The Cheat with the Ace of Diamonds* (1635) by Georges de La Tour
The Louvre Museum, Paris (France)
Math topic: Combinatorics and probability. Games of chance (Chapter 3, p. 45)

52. *The Dice Players* (ca. 1651) by Georges de La Tour
Preston Hall Museum, Stockton-on-Tees (Cleveland, UK)
Math topic: Combinatorics and probability. Games of chance (Chapter 3, p. 45)

53. *The Game of Knucklebones* (ca. 1734) by Jean-Baptiste-Siméon Chardin
The Baltimore Museum of Art (Baltimore, Maryland, USA)
Math topic: Combinatorics and probability. Games of chance (Chapter 3, p. 46)

54. *The Flemish Fair* by Marten van Cleve.
Musée des Beaux-Arts, Caen (France).
Math topic: Combinatorics and probability. Games of chance (Chapter 3, p. 45)

55. *The Worm Has Many Hearts* (1993–1994) by Alan Shields
KSU, Beach Museum of Art, Manhattan, Kansas (USA)
Math topic: Symmetry. Frieze pattern (Chapter 4, p. 67)

56. *The Sacrament of the Last Supper* (1955) by Salvador Dali
The National Gallery of Art (Washington, USA)
Math topic: Polyhedra (Chapter 2, p. 28)

57. *The Wamego House* (20th century) by George M. Kren
KSU, Beach Museum of Art, Manhattan, Kansas (USA)
Math topic: Linear perspective. Horizon. Vanishing points (Chapter 8, p. 133)

58. *The Wedding Feast at Cana* by Paolo Veronese
The Louvre Museum, Paris (France)
Math topic: Linear Perspective. Horizon (Chapter 8, p. 142)

59. *Three Throwsticks* (ca.1550–1069 BC). Egyptian Antiquities
The Louvre Museum, Paris (France)
Math topic: Combinatorics and probability. Games of chance (Chapter 3, p. 46)

60. *Young Boys Playing Dice in Front of Christiansborg Castle, Copenhagen, 1834* by Carl-Christian-Constantin Hansen
The Louvre Museum, Paris (France)
Math topic: Combinatorics and probability. Games of chance (Chapter 3, p. 46)

61. *3 Standard Stoppages* (1913) by Marcel Duchamp
The Museum of Modern Art, New York (USA)
Math topic: Combinatorics and probability. Randomness (Chapter 3, p. 46)

62. *Yellow Tree* (1948) by Roy Clinton Langford
KSU, Beach Museum of Art, Manhattan, Kansas (USA)
Math topic: Linear perspective. Horizon (Chapter 8, p. 132)

63. *21 with Cube* (2001) by Jesus (Jessie) Manuel Montes
KSU, Beach Museum of Art, Manhattan, Kansas (USA)
Math topic: Combinatorics and probability. (Chapter 3, p. 41; insert, pp. 40-41)

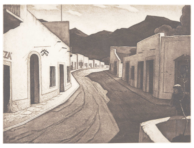
64. Title unknown (ca. 1955) by Roy Clinton Langford
KSU, Beach Museum of Art, Manhattan, Kansas (USA)
Math topic: Linear perspective. Horizon (Chapter 8, p. 131)

IMAGE INDEX

1. 

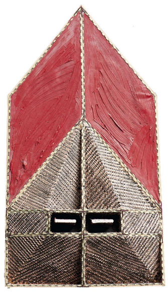
Mary Eck Holland Call (United States, 1893–1959)
Abandoned, 1950
Watercolor on paper
369 x 470 mm
KSU, Beach Museum of Art, 1951.3

Math topic: Linear perspective. Horizon (Chapter 8, p. 148))

2. 

Charles Merrick Capps (United States, 1898–1981)
Mexican Barber Shop, 1938
Aquatint on paper
215.9 x 292.1 mm
KSU, Beach Museum of Art, 1966.19

Math topic: Linear perspective. Vanishing points (Chapter 8, p. 149; insert, pp. 128-129)

3. 

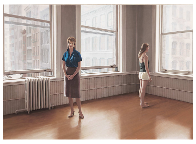
Jack Chevalier (United States, born 1948)
Red Groom, 1979
Balsa wood, paper, paint 38.1 x 21 x 6.4 cm
KSU, Beach Museum of Art, 1988.3

Math topic: Reflection symmetry (Chapter 4, p. 61; insert, pp. 16-17)

4. 

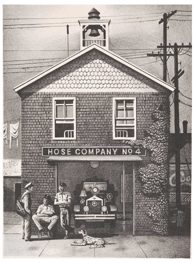
Carol Pylant (United States, born 1953)
No Masters, 1986
Oil on panel
76.2 x 66.99 cm
KSU, Beach Museum of Art, Friends of Art, 1988.6

Math topic: Linear perspective (Chapter 8, p. 127)

5. 

Carol Pylant (United States, born 1953)
Spring and Mercer, 1983-84
Oil on panel 100.01 x 146.05 cm
KSU, Beach Museum of Art, Friends of Art, 1988.9

Math topic: Two-point perspective (Chapter 8, p. 138)

6. 

Stevan Dohanos (United States, 1907–1994)
Hose Co #4, 1933
Lithograph on paper 349 x 260 mm
KSU, Beach Museum of Art, bequest of Raymond & Melba Budge, 1992.12. Art © Estate of Stevan Dohanos/Licensed by VAGA, New York, NY

Math topic: Linear perspective. Front view (Chapter 8, p. 136; insert, pp. 144-145)

7. 

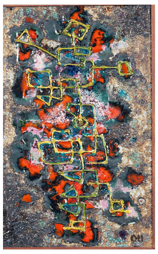
Orval F. Hempler (United States, 1915–1994)
Title unknown, 20th century
Glazed earthenware
50.8 x 29.21 cm
KSU, Beach Museum of Art, bequest of Orval F. Hempler Estate, 1994.21

Math topic: Reflection symmetry (Chapter 4, p. 62)

8. 

Orval F. Hempler (United States, 1915–1994)
Title unknown, 20th century
Glazed earthenware
41.28 x 50.8 cm
KSU, Beach Museum of Art, bequest of Orval F. Hempler Estate, 1994.23

Math topic: Circles (Chapter 5, p. 75)

9. 

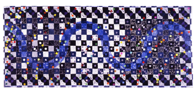
Orval F. Hempler (United States, 1915–1994)
Title unknown, 20th century
Glazed earthenware
48.9 x 28.58 cm
KSU, Beach Museum of Art, bequest of Orval F. Hempler Estate, 1994.32

Math topic: Topology. Tracing design (Chapter 10, p. 174)

10. 

Shirley Smith (United States, 1929–2013)
Kansas Gate Posts, 1985
Oil on canvas
129.54 x 185.42 cm
KSU, Beach Museum of Art, gift of Nancy Landon Kassebaum Baker, Burdick, Kansas, 1998.5

Math topic: Linear perspective. Front view (Chapter 8, p. 131; insert, pp. 128-129)

11. 

Alan Shields (United States, 1944–2005)
The Worm Has Many Hearts, 1993–1994
Acrylic, thread, wool yarn, and acrylic yarn on canvas
140.33 x 323.85 cm
KSU, Beach Museum of Art, Friends of the Beach Museum of Art purchase, 1999.14

Math topic: Symmetry. Frieze pattern (Chapter 4, p. 67)

12. 

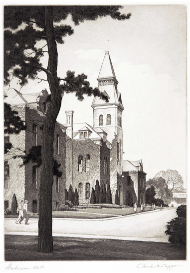
George M. Kren (United States, born Austria, 1926-2000)
The Wamego House, 20th century
Gelatin silver print
191 x 241 mm
KSU, Beach Museum of Art, gift of Margo Kren, 2001.35

Math topic: Linear perspective. Vanishing points (Chapter 8, p. 133)

13. 

Larry W. Schwarm (United States, born 1944)
Red Earth near Salamanca, Spain, May 2000, 2000
Chromogenic print, 211 x 565 mm
KSU, Beach Museum of Art, Friends of the Beach Museum of Art purchase, 2001.81

Math topic: Linear perspective. Horizon (Chapter 8, p. 135)

14. 

Charles Merrick Capps (United States, 1898–1981)
Anderson Hall, 1947
Lithograph on paper
219 x 156 mm
KSU, Beach Museum of Art, gift of Elizabeth Taggart, 2002.291

Math topic: One-point perspective (Chapter 8, p. 137)

15. 

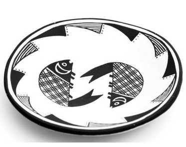
Jesus (Jessie) Manuel Montes (United States, born Mexico, 1935–2013)
21 with Cube, 2001
Corrugated paperboard with acrylic
PEDESTAL 79.8 x 41.4 x 41.8 cm
CUBE 50 x 50.3 x 50.4 cm
KSU, Beach Museum of Art, Friends of the Beach Museum of Art purchase, 2003.5

Math topic: Combinatorics and probability (Chapter 3, p. 41; insert, pp. 48-49)

16. 

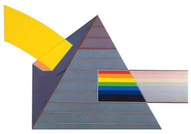
Norma Bassett Hall (United States, 1889–1957)
Autumn Road, ca. 1928
Color woodcut on paper
165 x 218 mm
KSU, Beach Museum of Art, John F. Helm, Jr. Memorial Fund, 2004.3

Math topic: Linear perspective. Vanishing points (Chapter 8, p. 149)

17. 

Delores Lewis Garcia (United States, Acoma Pueblo, born 1938)
Plate, 20th century
Glazed earthenware
127 mm x 25.4 mm
KSU, Beach Museum of Art, gift of Jim, Angele, Luke, & Julia Johnson, in memory of Jeaneane Berryhill Johnson, 2004.323

Math topic: Rotational symmetry (Chapter 4, p. 63)

18. 

Roger Lane Routson (United States, born 1951)
Konza Prism, 1984
Acrylic on shaped canvas over panel
106.68 x 205.74 x 5.08 cm
KSU, Beach Museum of Art, gift of James R. Ward, 2004.80

Math topic: Geometrical Optics. Refraction (Chapter 7, p. 109)

19. 

Alan Shields (United States, 1944–2005)
Kansas Meatball, 1985–1998
Acrylic on canvas over aluminum tubing, bolts and nuts, and cotton thread
292.1 x 330.2 cm
INNER BALL 243.9 x 223.5 cm
KSU, Beach Museum of Art, Alan Shields Memorial Fund, Ross and Marianna Kistler Beach Endowment, and gift from the Estate of William Salero, 2007.111

Math topic: Polyhedra. Icosachedron (Chapter 2, p. 25)

20. 

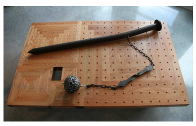
Roy Clinton Langford (United States, 1903–1990)
Title unknown, ca. 1955
Watercolor with graphite on paper
251 x 354 mm
KSU, Beach Museum of Art, The Roy C. Langford Collection, gift of the Langford family, 2008.367

Math topic: Linear perspective. Horizon (Chapter 8, p. 131)

21. 

Roy Clinton Langford (United States, 1903–1990)
Yellow Tree, 1948
Watercolor and gouache with graphite on paper
356 x 537 mm
KSU, Beach Museum of Art, The Roy C. Langford Collection, gift of the Langford family, 2008.388

Math topic: Linear perspective. Horizon (Chapter 8, p. 132)

22. 

John L. Vogt (United States, 1930–2009)
Pyrrho Techniko, 1989
Wood and steel
48.26 x 99.06 x 160.02 cm
KSU, Beach Museum of Art, gift of Mrs. John L. Vogt, Mark Vogt, and Jamie Kitch, 2009.116

Math topic: Fibonacci numbers (Chapter 6, p. 89; insert, pp. 96-97)

23. 

Tony Da (United States, San Ildefonso Pueblo, 1940–2008)
Plate, 20th century
Glazed and incised earthenware
2.9 x 16.2 x 16.2 cm
KSU, Beach Museum of Art, gift of Mel and Mary Cottom, 2012.182

Math topic: Symmetry (Chapter 4, p. 63)

24. 

Naomi Gutierrez (United States, Santa Clara Pueblo)
Blackware jar, 20th century
Glazed earthenware
28 x 32 x 32 mm
KSU, Beach Museum of Art, gift of Mel and Mary Cotton, 2012.208

Math topic: Symmetry. Frieze pattern (Chapter 4, p. 65)

25. 

Shirley Duran (United States, Santa Clara Pueblo)
Seed pot, 20th century
Glazed earthenware
1.7 x 1.7 x 1.4 cm
KSU, Beach Museum of Art, gift of Mel and Mary Cotton, 2012.216

Math topic: Symmetry. Frieze pattern (Chapter 4, p. 65)

26. 

Florence Yepa (United States, Jemez Pueblo, born 1949)
Seed pot, 20th century
Glazed earthenware
2.22 x 3.8 cm
KSU, Beach Museum of Art, gift of Mel and Mary Cotton, 2012.218

Math topic: Symmetry. Frieze pattern (Chapter 4, p. 66)

27. 

Maria Margarita (Margaret) Tafoya (United States, Santa Clara Pueblo, 1904–2001)
Bowl, 20th century
Glazed earthenware
94 mm x 161 mm
KSU, Beach Museum of Art, gift of Mel and Mary Cotton, 2012.226

Math topic: Symmetry. Frieze pattern (Chapter 4, p. 64)

28. 

Antonín Rückl and Sons, Ltd., Glassworks
(Czech Republic)
Bowl, late 20th century
Lead crystal
12.7 x 21.59 x 21.59 cm
KSU Collection, gift of Joseph & Elizabeth Barton-Dobenín, U1.1998

Math topic: Symmetry. Frieze pattern (Chapter 4, p. 66)

29. 

Antonín Rückl and Sons, Ltd., Glassworks
(Czech Republic)
Vase, late 20th century
Lead crystal
29.21 x 13.97 x 13.97 cm
KSU Collection, gift of Joseph & Elizabeth Barton-Dobeín, U2.1998

Math topic: Symmetry. Frieze pattern (Chapter 4, p. 67)

30. 

Antonín Rückl and Sons, Ltd., Glassworks
(Czech Republic)
Vase, late 20th century
Lead crystal
30.48 x 12.7 x 12.7 cm
KSU Collection, gift of Joseph & Elizabeth Barton-Dobenín, U4.1998

Math topic: Star Polygons (Chapter 9, p. 155)

31. 

Mitsugi Ohno (United States, born Japan, 1926–1999)
Klein Bottle
Blown glass
21.59 x 12.07 cm
KSU, Beach Museum of Art, gift of William L. Richter in memory of Mitsugi Ohno, U4.2009

Math topic: Topology (Chapter 10, p. 165; insert, pp. 176-177)

NAME INDEX

A
Arnold, Vladimir Igorevich 82
Arp, Jean 46
Attard, Norbert Francis 100

B
Barbari, Jacopo de' 28
Beach, Marianna 17
Beach, Ross 17
Bennett, Alan 169
Bézout, Étienne 162
Blumenfeld, W. 143
Burckel, Robert 2, 11

C
Call, Mary Eck Holland 148
Capps, Charles Merrick 137, 143
Caravaggio, Michelangelo Merisi da 45
Cardwell, Alvin B. 169
Cézanne, Paul 46
Chardin, Jean-Baptiste-Siméon 46
Chevalier, Jack 61, 71
Cicero, Marcus Tullius 76
Cleve, Marten van 45
Cutler, William G. 15
Curry, John Steuart 17

D
Da, Tony 63
Dali, Salvador 28
Dante, Alighieri 145
Delaunay, Robert 77
Delaunay, Sonia 77
Dohanos, Stevan 136, 144
Duchamp, Marcel 46
Duke, Linda 11
Duran, Shirley 65
Dürer, Albrecht 28

E
Eco, Umberto 94, 96, 97
Escher, Maurits Cornelis 19, 28, 169
Exekias, 45

F
Falbo, Clement 94–96
Farrell, Francis 17
Fermat, Pierre de 43, 44, 110, 117
Fuller, Richard Buckminster 29

G
Gardner, Martin 94–96
Giovanni da Verona 28
Glasgow, Linda 11
Gombaud, Antoine (le Chevalier de Méré, M.) 43
Goldsworthy, Andy 77
Grimshaw, Nicholas 100
Gutierrez, Naomi 65, 197

H
Hall, Leon 85
Hall, Norma Bassett 144, 149
Hanning, Gérald 100
Hansen, Carl-Christian-Constantin 46
Hardie, George 113
Helm, John 17
Helmholtz, H. 143
Hempler, Orval F. 62, 75, 76, 174
Herz-Fischer, Roger 95
Hillebrand, F. 143
Holbein the Younger, Hans 151
Hooch, Pieter de 45

I
Inuzuka, Sadashi 77

J
James, Linda 146

Jeanneret, Charles Edouard
(Le Corbusier) 20, 98–100
Jean, Roger V. 98

K

Kandinsky, Wassily 77, 84
Kelly, Ellsworth 77
Klein, Christian Felix 165
Kren, George M. 133, 144
Kren, Margo 145
Ketterer, Theresa Marie 11

L

Landon Kassebaum Baker,
Nancy 148
Langford, Roy Clinton 131, 132, 145
La Tour, Georges de 45
Leonardo da Vinci 28, 94, 96
Le Corbusier (Jeanneret, Charles
Edouard) 20, 98–100
LeWitt, Sol 21, 77
Long, Richard 77
Luneburg, R. K. 143

M

Malevich, Kasimir 77
Markowsky, George 94–96
Merz, Mario 20, 100
Montes, Jesus (Jessie) Manuel 41, 44
Morellet, François 47
Möbius, August Ferdinand 165

Noland, Kenneth 77

O

Ohno, Mitsugi 165, 168

P

Pacioli, Luca 28
Parks, Gordon 17
Pascal, Blaise 43, 44, 103, 107, 110
Picasso, Pablo 46
Price, Sarah 11
Pylant, Carol 127, 138, 145

R

Rozhkovskaya, Tamara 12
Routson, Roger Lane 109, 113
Rückl, Antonín 158
Rückl, Sebastian 158

S

Samushia, Lado 11
Sandzen, Birger 17
Schwarm, Larry W. 135, 147
Seaton, Elizabeth 11
Shields, Alan 25, 27, 67
Schlageck, Kathrine 11, 17, 22
Sidorova, Nadia 11, 52
Slagg Winifred N. 15
Smith, Shirley 131, 147
Snellius, Willebrord 114
Stroh, Charles 17, 89

T

Tafoya, Maria Margarita
(Margaret) 64
Taton, René 100
Thayer, Eli 14

V

Vermeer, Johannes 147
Veronese, Paolo 142
Vogt, John L. 89, 92

W

Wagon, Stan 85
Wefald, Ruth Ann 17

Y

Yepa, Florence 66

Z

Zharkov, Ilia 11
Zharkov, Styopa 11
Zorio, Gilberto 77
Zybach, Frank 86

SUBJECT INDEX

A
Angle
– of a regular polyhedron 31
– of a regular triangle 31
– of incidence
– of reflection 117, 125
– of refraction 111
Art project,
concentric circles 84
decorated die 53
decorated plate 159
decorated vase 74
Paper People Chain 74
paper plate icosahedron 36
rainbow paper collage 120
Random Person 54
snake of dice 52
string star polygon 160
toothpick icodahedron 39
vase drawing 159

B
Bézout lemma 162
Bohemian crystal 155
Bowl
– blackware 64
– lead crystal 66

C
Circles 75
Circle, chord 86
diameter 86
secant line 86
tangent line 87
inscribed angle 87
arc 87
Closed formula 91
Combinatorics 41
– enumerative 21
Cube 24, 30

D
Degree
– of a design vertex 175
Design traceable 175
Die 42, 47
– left-handed 42, 47, 48
– right-handed 42, 47, 48, 57
Dodecahedron 27, 30

E
Euclidean algorithm 157
Edge
– of a design 175
– of a polyhedron 27

F
Face
– of a polyhedron 26
Fermat's principle 110, 117
Fibonacci numbers 20, 23, 89, 91
Frieze pattern 64
– dizzy hop 68, 70, 71
– dizzy jump 68, 70, 71
– dizzy slide 68, 70, 71
– hop 68, 70, 71
– jump 68, 70, 71
– step 68, 70, 71
– slide 68, 70, 71

G
Game, card 45
Game of chance 45
Golden ratio 90, 92
Golden rectangle 95
Golden spiral 95
Greatest common divisor 157, 162

H
Hexagon 31, 33

Horizon 128
Hypotenuse 80

I

Index of refraction 115
Icosahedron 26–30, 35

K

Klein bottle 165
Knot 176
– figure-eight 176, 178
– trefoil 176, 178
– trivial 176, 178

L

Light ray 110
– bent 122
– incoming 122
– laser 120
– refracted 110
– reflected 116
Light
– white 111
Light refraction 110
Light travel 110
Light wave 114

M

Method
– of linear perspective 128
– of mathematical induction 107
Modulor 20, 98
Möbius band 165

O

Octahedron 27, 30
Optics geometrical 110
Ornament 159
– star 155

P

Pascal triangle 103, 107
Pentagon 31, 100
Perspective, linear 127
– one-point 137
– two-point 137
– multi-point 138
Pithagorean theorem 101, 105
Platonic solid 27, 30
Point
– vanishing 132
Polyhedron 26
– convex 27
– dual 34
– regular 27
Polygon
– regular 31, 39
– star 156
Pot, seed 65, 66
Prairie 14, 110
Prism 109, 112
Pueblo pottery 72
Probability 42
Problem of Points 44

R

Rainbow 110, 120
Randomness 46
Reccurence 91, 105
Reflection symmetry 62
Rhombicuboctahedron 28
Reflection principle 117
Reflection
– of light 117
– total internal 116
Refraction 110

Q

Quadratic equation 105

S

Surface
– nonorientable 165
– – closed 167
Snell's law 111, 113
Spectrum 111
– of white light 111
Sphere 29
Sunlight 111, 122
Symmetry 61
– rotational 64
– tanslational 69
Symnmetry, reflection 62
– glid 70
– horizontal 69
– vertical 69

T

Tetrahedron 27, 30
Topological project,
Möbius bands 170
Klein bottles and tori 170
Topology 165, 172
Triangle 26, 105

– equilateral 102
– isosceles 39
– right 80, 123
– regular 26, 31
Tracing design 174
Torus 166

V

Vase 64, 74,
– lead crystal 67, 155, 158,
Velocity, light 114
Vertex
– of design 175
– of a die 42
– of a polyhedron 27
– of a star polygon 155
– through 175

U

Unknot 176